

Bending Moment and Shear Force Response at the Bottom of Circular Steel Tall Silo in Wind Environment

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Abstract

Failures of wind engineering in the past reveals that even strict compliance to conventional design methods, including accepted codes and standards, was not sufficient to prevent failures. No wind loading standard or code, can claim to cover completely wind loading. Wind forces are responsible for huge destruction in the world. The present research was on the study of circular steel silo subjected to rigorous wind loading and its effects on the anchorage requirements of tall silos. For the analysis of such a situation, there were few techniques available namely, finite element method, structural matrix analysis. The conventional methods of analysis gives more conservative results and hence gives penalty to the clients. This paper reports the comparison of results by various analysis of tall silos for additional bending moment and shear force in wind environment. The maximum bending moment values (Nm E+06) for silos with H/D ratio of 7.5 in static, quasi-static (IS code), quasi-static (AS code), dynamic and random analysis techniques were 39.0, 68.3, 185.0, 40.2 and 63.1 respectively. The maximum shear force values (N E+05) for silos with H/D ratio of 7.5 in static analysis, quasi-static (IS code), quasi-static (AS code), dynamic and random analysis techniques were 16.1, 28.1, 76.3, 16.6 and 26.0 respectively. The present analysis is on the basis of wind speed data around Delhi, but the model can be used for any wind speed and location with minor changes.

Key words : Wind excitation, Response spectrum, Shear force and bending moment, Probability density function, Aspect ratio.

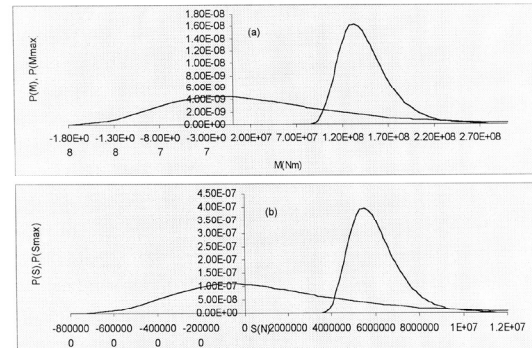
Due to rapid advancement in the materials and construction techniques, nowadays tall, light weight and flexible structures are being constructed. Such structures generally exhibit an increased susceptibility to the action of winds. The most common lateral load is a wind load. Wind loads vary around the world. Meteorological data collected by various national weather services are one of the most reliable sources of wind data. Factors that effect the wind load include the geographic location, elevation, degree of exposure, relationship to nearby structures, building structural materials, height and size, direction of prevailing winds, velocity of prevailing winds, and positive or negative pressures due to design features. All of these factors are taken into account when the lateral loads on the structure are calculated. It is often necessary to examine more than one wind load case. Therefore while designing these structures, it is to be ensured that the

performance of structures subjected to action of wind will be adequate during their anticipated life from viewpoint of both structural safety and serviceability. To achieve these objectives, it is desirable to improve the understanding of the behavior of the wind environment on structures with an emphasis on more precise definition of wind forces for structural design. Major dynamic effects of wind on structures are forced vibrations and self excited vibrations. Wind forces act as an external forces and their magnitude depends upon area of the exposure and the shape of the structure unlike earthquake forces which act as inertia forces on account of earthquake motion of the ground, their magnitude depending upon the mass of the structure. The conventional analysis and design oversimplifies the requirements of special treatment for non-conventional structures. The conventional procedure of design and analysis includes the assumption of wind load as static or quasi-static. In

Table 1. First three natural frequencies (Hz) for tall silos.

| Diameter (m) | 12.0 | 10.5 | 9.0 | 7.5 | 6.0 |
|------------------|-------|-------|------|------|------|
| Height (m) | 45.0 | 45.0 | 45.0 | 45.0 | 45.0 |
| First frequency | 1.80 | 1.69 | 1.64 | 1.57 | 1.45 |
| Second frequency | 15.69 | 10.55 | 9.40 | 8.39 | 7.13 |
| Third frequency | 88.7 | 77.5 | 66.2 | 55.5 | 44.7 |

peak wind approach ; depending upon parameters such as risk level, local topography, terrain roughness, height and size of the silo, design wind pressure is determined. From value of design wind pressure, wind load is determined using pressure coefficients and frontal area of the silo. For using mean wind approach, besides the parameters required for peak wind approach (1), value of gust factor is required for which fundamental natural frequency of the structure is required to be estimated. This was not applicable for highly flexible thin walled structures subjected to no internal pressure, e.g. silos during empty condition and unfavorable climatic conditions. The wind load was taken as uniformly distributed or considered as triangular linear variation or even trapezoidal. This concept was not a valid one for thin walled structures such as surface type silo especially made of thin steel walls. To arrive at the thickness of steel wall, economic considerations required the estimation of it by thin cylinder theory and dynamic grain pressure theory without incorporating wind load. This called for the spatial behavior of the slender structures in the vicinity of other structures by a random wind loading. This enabled the automatic treatment of vibrations effects on the outer face of the structure. The anchorage requirement for the empty silos considering wind load effects was analyzed (2). They adopted the quasi-static wind load approach. This had got limitations when the wind load had a random nature and the aerodynamic characteristics of structure were not considered. The wind load effect through a mathematical model was analyzed to include the equivalent wind spectrum technique considering the schematization of the wind as a stochastic stationary Gaussian process (3). However the mathematical technique adopted in the research was a closed form approach. The paper considered in an elaborate manner the probability den-

**Figure 1.** (a) PDF of maximum base bending moment ; (b) PDF of maximum base shear force for tall silo.

sity function for the wind response of the structures. The significance of this study was the advantage of simultaneous treatment of both vertical and horizontal portions of the structures and this approach gave importance to the anchorage requirement.

Methods

In the present case five circular steel silos having height to diameter ratio 3.75, 4.28, 5.0, 6.0 and 7.5 with a single height of 45 m silo subjected to the condition of height to diameter ratio more than 3.0 were selected as this ratio represented a workable H/D ratio (aspect ratio) in this region. This choice was based on the study reported Briassoulis and Packnold (2). The concept of dynamic grain pressure during the filling and emptying process was also based on the very well established Janssen theory for calculating the wall thickness in preliminary design (4). The vertical pressure q and horizontal pressure p at a depth y are given by the equations ;

$$q = (\gamma R / k \mu') (1 - e^{-k \mu' y / R})$$

$$p = kq$$

where R is hydraulic radius (area of cross-section divided by the perimeter) of the silo. The buckling critical stress (5) in the wall was estimated for axial compression without hoop tension by

$$f_{cr} = (C_c E t) / r$$

Table 2. Values of maximum base BM (Nm E + 06) for tall silos by various analysis.

| Height | Dia | H/D | Static | Qstat-I | Qstat-A | Dynamic | Random |
|--------|-------|------|----------|----------|----------|----------|----------|
| 45.00 | 12.00 | 3.75 | 7.81E+01 | 1.63E+02 | 3.79E+02 | 7.94E+01 | 1.33E+02 |
| 45.00 | 10.50 | 4.28 | 6.82E+01 | 1.43E+02 | 3.31E+02 | 7.00E+01 | 1.16E+02 |
| 45.00 | 9.00 | 5.00 | 5.85E+01 | 1.23E+02 | 2.81E+02 | 6.04E+01 | 9.78E+01 |
| 45.00 | 7.50 | 6.00 | 4.84E+01 | 1.02E+02 | 2.34E+02 | 5.03E+01 | 8.03E+01 |
| 45.00 | 6.00 | 7.50 | 3.90E+01 | 6.83E+01 | 1.85E+02 | 4.02E+01 | 6.31E+01 |

where

$$C_0 = 0.374 / (\text{SQRT}(1.0 + (r/100t)))$$

for $r/t \leq 212$

$$C_0 = 0.315 / (\text{SQRT}(0.1 + (r/100t)))$$

for $r/t > 212$

If f_{cr} computed from above exceeds $3/8 f_y$, then

$$f_{cr} = f_y (1.0 - 0.347 (f_y / f_{cro})^{0.6})$$

where f_{cro} is the f_{cr} calculated from above equation.

Assuming $f_y = 420 \text{ N/mm}^2$, a check on the adequacy of wall thickness was applied taking advantage of the hoop tension using equations :

The axial compression with hoop tension as given by

$$f_{ch} = (C_h E t) / r \leq 3/8 f_y$$

$$C_h = C_0 + (0.45 - C_0) (\rho / (\rho + 0.007))$$

where

$$\rho = (p/E) (r/t)^{1.5}$$

in which p is the internal lateral pressure in the bin. For design, allowable stress = critical stress/1.5 >

maximum of hoop or meridional stress. Wind loads not only exert a constant oscillating force, but also increases as the height of the structure increases. The wall thickness was not only going to be parameter to counter the wind load effects but the proper anchorage at the base against overturning of the top about the base. This work was based particularly on this background. For the purpose of computations, the steel of the cylindrical shell has a modulus of elasticity $E = 200 \text{ Gpa}$, poisson's ratio of 0.30, and mass density of $7,900 \text{ kg/m}^3$. The high-frequency base technique is applicable to the large majority of tall structures and has the advantage of allowing the structural engineer to easily update the results in the case of a change in the structure's stiffness, mass or natural frequency. The traditional techniques are approximate and do not accurately represent the correct distribution of loads, but are cheaper and less complex. Several computer programs were prepared by the author for using this technique.

The approaches for the analysis of such structures under static, quasi-static were even dynamic (wind totally dynamic) response needed the thorough modification to include the random wind loading. The random wind loading could be accounted by a spectral loading with probability analysis to find the exact response. But then when the shell was empty and the wind load was extensive what was going to happen to the wall and at the base. Wind loading will be effective and severe when it will be empty, so the wind load under the circumstances was considered

Table 3. Values of maximum base shear force (N E+03) for tall silo by various analysis.

| Height | Dia | H/D | Static | Qstat-I | Qstat-A | Dynamic | Random |
|--------|-------|------|----------|----------|----------|----------|----------|
| 45.00 | 12.00 | 3.75 | 3.22E+03 | 6.73E+03 | 1.56E+04 | 3.27E+03 | 5.50E+03 |
| 45.00 | 10.50 | 4.28 | 2.81E+03 | 5.88E+03 | 1.36E+04 | 2.89E+03 | 4.76E+03 |
| 45.00 | 9.00 | 5.00 | 2.41E+03 | 5.06E+03 | 1.16E+04 | 2.49E+03 | 4.03E+03 |
| 45.00 | 7.50 | 6.00 | 2.00E+03 | 4.21E+03 | 9.63E+03 | 2.07E+03 | 3.31E+03 |
| 45.00 | 6.00 | 7.50 | 1.61E+03 | 2.81E+03 | 7.63E+03 | 1.66E+03 | 2.60E+03 |

to be critical. In view of this the spectral analysis technique, which considered the response of random nature, was of value in this study. The study of wind effect was of paramount importance especially for the proper anchorage requirements of designed to stabilize the entire structure against overturning. The entire structure was assumed to be mathematically lumped at three places namely lower, middle and top. After inverting the stiffness matrix the flexibility matrix was formed for conjugate beam method. Such analysis was quite common in the stress analysis of structures. With the use of this matrix, first three natural frequencies (Table 1) were obtained. For random analysis, the wind velocity was divided into two parts namely, static part and fluctuating part. For the fluctuating part the basic mathematical wind model developed by Sharma (5) was followed for spectral analysis in this study. Relation between reduced frequency and energy was given as

$$X = \log(nz/V)$$

$$Y = \log n(Sn)$$

$$Y = -1.991225 - 0.4447708 X - 0.1091633 X^2$$

Relation between reduced frequency and normalized energy

$$X = \log(nz/V)$$

$$Y_1 = \log n(Sn)/\sigma^2$$

$$Y_1 = -1.531011 - 0.2919729 X - 0.1144482 X^2$$

The mean square value of the fluctuating component of the response was calculated (6) following the standard procedure given in AS : 1170 (part-2) 1989.

$$x^2 = 4(F^2/V^2k^2) \int_0^8 \chi^2(n\sqrt{A}/V) |H(n)|^2 u^2 S_u^*(n) dn$$

Where, F/K is the static displacement, $\chi^2(n)$ = aerodynamic admittance function, $|H(n)|^2$ = mechanical admittance, n = forcing frequency, u^2 = Mean square fluctuating velocity, $S_u^*(n)$ = power spectral density of fluctuating velocity, \sqrt{A} = characteristic dimension.

The static part of the response due to the mean component of wind loading was calculated (7) as per IS : 875 (Part-3) 1987. In regard to the consideration of a time series effect, one-hour interval time was adopted as proposed by Davenport (8). For probability analysis, extreme value type-1 distribution, probability distribution function curve were plotted in Figure 1 for base BM and base SF. The brief procedure is given below :

$$\alpha = 1.282/\sigma_y$$

Where σ_y = standard deviation, α = measure of dispersion

$$u = m - 0.577/\alpha$$

Where m = mean value, u = mode of the distribution

$$y = u + w/\alpha$$

Where y is the variable for which PDF is to be plotted. Corresponding to arbitrary values of w , values of $F_w(w)$ are available in the standard tables.

$$\text{Then } F_y(y) = \alpha \cdot F_w(w)$$

and y can be found from above corresponding to every w .

Hence we can plot a curve between y and $F_y(y)$ which is the required probability distribution function curve for variable y .

For the distribution of largest values the mean value factor of maxima is given by

$$\eta_{(\max)} = \sqrt{(2 \log_e vT) + 0.577} / (\sqrt{(2 \log_e vT)})$$

and the standard deviation factor is given by

$$\sigma_{(\eta \max)} = (\pi/\sqrt{6}) (1/(\sqrt{(2 \log_e vT)}))$$

where v , the crossing rate is given by

$$v = \sqrt{\int_0^{\infty} n^2 S(n) dn / \int_0^{\infty} S(n) dn}$$

and the computer program was developed accordingly.

Results and Discussion

Comparison of Bending Moment for Tall Silos

Results of the maximum bending moment for tall silos, given in Table 2 shows that corresponding to the maximum bending moment at the base by static analysis, the maximum bending moment at the base by quasi-static analysis using Indian code, quasi-static analysis using Australian code, dynamic analysis and random analysis are 34.68, 92.19, 39.6 and 70.81% higher respectively for silo having height of 45 and 12 m diameter. The corresponding values were 34.91, 97.23, 40.43 and 70.09% higher for 45 m height and 10.5 m diameter, 34.88, 98.44, 40.08 and 67.18% higher for 45 m height and 9 m diameter, 35.71, 100.0, 40.47 and 65.91% higher for 45 m height and 7.5 m diameter and 34.36, 98.29, 39.74 and 61.79% higher for 45 m height and 6 m diameter.

In static analysis the maximum bending moment at the base observed for silo having 45 m height and 10.5 m diameter was 12.68% lower than the maximum bending moment at the base observed for silo having 45 m height and 12 m diameter. It was 14.22% lower for silo having 45 m height and 9 m diameter than for silo having 45 m height and 10.5 m diameter, 17.26% lower for silo having 45 m height and 7.5 m diameter than for silo having 45 m height and 9 m diameter, 19.42% lower for silo having 45 m height and 6 m diameter than for silo having 45 m height and 7.5 m diameter.

In quasi-static analysis using Indian code the maximum bending moment at the base observed for silo having 45 m height and 10.5 m diameter was 12.27% lower than the maximum bending moment at the base observed for silo having 45 m height and 12 m diameter, 13.97% lower for silo having 45 m height and 9 m diameter than for silo having 45 m height and 10.5 m diameter, 17.07% lower for silo having 45 m height and 7.5 m diameter than for silo having 45 m height and 9 m diameter, 33.04% lower for silo having 45 m height and 6 m diameter than for silo having 45 m height and 7.5 m diameter.

In quasi-static analysis using Australian code the maximum bending moment at the base observed was 12.66% lower for silo having 45 m height and 10.5 m diameter than for silo having 45 m height and 12 m diameter, 15.10% lower for silo having 45 m height and 9 m diameter than for silo having 45 m height and 10.5 m diameter, 16.72% lower for silo having 45 m height and 7.5 m diameter than for silo having 45 m height and 9 m diameter, 20.94% lower for

silo having 45 m height and 6 m diameter than for silo having 45 m height and 7.5 m diameter.

In dynamic analysis the maximum bending moment at the base observed for silo having 45 m height and 10.5 m diameter was 11.84% lower than the maximum bending moment at the base observed for silo having 45 m height and 12 m diameter, 13.43% lower for silo having 45 m height and 9 m diameter for silo having 45 m height and 10.5 m diameter, 16.72% lower for silo having 45 m height and 7.5 m diameter than for silo having 45 m height and 9 m diameter, 20.08% lower for silo having 45 m height and 6 m diameter than for silo having 45 m height and 7.5 m diameter.

In random analysis the maximum bending moment at the base observed for silo having 45 m height and 10.5 m diameter was 12.78% lower than the maximum bending moment at the base observed for silo having 45 m height and 12 m diameter, 15.69% lower for silo having 45 m height and 9 m diameter than for silo having 45 m height and 10.5 m diameter, 17.89% lower for silo having 45 m height and 7.5 m diameter than for silo having 45 m height and 9 m diameter, 21.42% lower for silo having 45 m height and 6 m diameter than for silo having 45 m height and 7.5 m diameter.

Comparison of Shear Force for Tall Silos

Results of the maximum shear force for tall silos (Table 3) show that corresponding to the maximum shear force at the base by static analysis, the shear force at the base by quasi-static analysis using Indian code, quasi-static analysis using Australian code, dynamic analysis and random analysis are 33.98, 91.29, 39.16 and 70.21% higher respectively for silo having height 45 m and 12 m diameter. The corresponding values were 34.91, 97.23, 40.43, and 69.39% higher for 45 m height and 10.5 m diameter, 34.88, 98.44, 40.08 and 67.22% higher for 45 m height and 9 m diameter, 35.71, 100.0, 40.47 and 65.50% higher for 45 m height and 7.5 m diameter, 34.53, 98.29, 39.74 and 61.49% higher for 45 m height and 6 m diameter.

In static analysis the maximum shear force observed at the base for silo having 45 m height and 10.5 m diameter was 12.73% lower than the maximum shear force at the base for silo having 45 m height

and 12 m diameter. It was 14.24% lower for silo having 45 m height and 9 m diameter than for silo having 45 m height and 10.5 m diameter, 17.01% lower for silo having 45 m height and 7.5 m diameter than for silo having 45 m height and 9 m diameter, 19.50% lower for silo having 45 m height and 6 m diameter than for silo having 45 m height and 7.5 m diameter.

In quasi-static analysis using Indian code the maximum shear force observed at the base for silo having 45 m height and 10.5 m diameter was 12.62% lower than the maximum shear force at the base for silo having 45 m height and 12 m diameter, 13.94% lower for silo having 45 m height and 9 m diameter than for silo having 45 m height and 10.5 m diameter, 16.80% lower for silo having 45 m height and 7.5 m diameter than for silo having 45 m height and 9 m diameter, 33.25% lower for silo having 45 m height and 6 m diameter than for silo having 45 m height and 7.5 m diameter.

In quasi-static analysis using Australian code the maximum shear force observed at the base for silo having 45 m height and 10.5 m diameter was 12.82% lower than the maximum shear force at the base for silo having 45 m height and 12 m diameter, 14.70% lower for silo having 45 m height and 9 m diameter than for silo having 45 m height and 10.5 m diameter, 16.98% lower for silo having 45 m height and 7.5 m diameter than for silo having 45 m height and 9 m diameter, 20.77% lower for silo having 45 m height and 6 m diameter than for silo having 45 m height and 7.5 m diameter.

In dynamic analysis the maximum shear force observed at the base for silo having 45 m height and 10.5 m diameter was 11.62% lower than the maximum shear force at the base for silo having 45 m height and 12 m diameter, 13.84% lower for silo having 45 m height and 9 m diameter than for silo having 45 m height and 10.5 m diameter, 16.87% lower for silo having 45 m height and 7.5 m diameter than for silo having 45 m height and 9 m diameter, 19.81% lower for silo having 45 m height and 6 m diameter than for silo having 45 m height and 7.5 m diameter.

In random analysis the maximum shear force observed at the base for silo having 45 m height and 10.5 m diameter was 13.45% lower than the maximum shear force at the base for silo having 45 m height and 12 m diameter, 15.33% lower for silo having 45 m height and 9 m diameter than for silo having 45 m

height and 10.5 m diameter, 17.87% lower for silo having 45 m height and 7.5 m diameter than for silo having 45 m height and 9 m diameter, 21.45% lower for silo having 45 m height and 6 m diameter than for silo having 45 m height and 7.5 m diameter.

Conclusion

The tall silos investigated had H/D ratio of 3.75, 4.28, 5.0, 6.0 and 7.5. The maximum bending moment values (Nm E + 06) for silos with H/D ratio of 7.5 in static, quasi-static (IS code), quasi-static (AS code), dynamic and random analysis techniques were 39.0, 68.3, 185.0, 40.2 and 63.1 respectively. The values of maximum bending moment at the base for other H/D ratios exhibited similar trend.

The maximum shear force values (N E + 05) for silos with H/D ratio of 7.5 in static analysis, quasi-static (IS code), quasi-static (AS code), dynamic and random analysis techniques were 16.1, 28.1, 76.3, 16.6 and 26.0 respectively. The values of maximum shear force at the base for other H/D ratios exhibited this trend. For tall silos height, there was gradual decrease in B. M and S. F values with the increase in height to diameter ratio from 3.75 to 7.5.

Based on the present investigation following conclusions are drawn : In the power spectral density function of the wind, the low frequency component is predominant. For the silos with natural frequencies less than 1 Hz, the dynamic effect of the wind is more critical as compared to its static effect. Extreme value statistics enable computation of the most probable peak values of the structural response. The bending moment and shear force response of a silo structure to the random wind forces can be represented fairly accurately considering only the first two modes of vibrations which causes additional design values. The response contribution from fluctuating component of wind is dominating for tall silos with H/D ratio greater than 3.0. Hence random analysis is strongly recommended to ensure safety at the lowest cost of the clients.

References

1. Singh J., M. L. Bansal and V. R. Sharma. 2008. Peak and mean wind approach—a case study of silos. Proc. Int. Conf., INSHAB CIT, Coimbatore, India.
2. Briassoulisa D. and D. A. Packnold. 1986. Anchorage

- requirements for wind loaded empty silos. *J. Struct. Engg. ASCE* 112 : 308—324.
3. Solari. 1988. Equivalent wind spectrum technique : Theory and applications. *J. Struct. Engg. ASCE* 114 : 1303—1323.
 4. Janssen H. A. 1895. On the pressure of grain in silos. *Inst. Civil Enggs., London* 124 : 553—555.
 5. Sharma V. R. 1993. Spectral characteristics and extreme value analysis for winds in India and codal provisions. Doctoral thesis, volumes I & II, IIT, New Delhi, India.
 6. AS 1170.2—1889, SAA loading code, part 2 : Wind loads. Standard Assoc. Australia.
 7. IS : 875 (part-3) Indian Standard Code of practice for design loads (other than earthquake for buildings and structures). New Delhi, India.
 8. Davenport A. G. 1964. Note on the distribution of the largest value of a random function with application to gust loading. *Proc. 28, Inst. Civil Enggs. London*, pp. 187—196.