

## **Effect on Bending Moment and Shear Force at the Bottom of Circular Steel Medium Silo in Wind Environment**

M. L. BANSAL<sup>1</sup>, JASPAL SINGH<sup>2</sup> AND V. R. SHARMA<sup>2</sup>

<sup>1</sup>*College of Agricultural Engineering & Technology, CCS Haryana Agricultural University  
Hisar 123001, India*

<sup>2</sup>*College of Agricultural Engineering, Punjab Agricultural University  
Ludhiana 141004, India  
E-mail : mlbns12001@yahoo.co.in*

### **Abstract**

In many parts of the world, due to wind, huge damage occurs every year. Even strict compliance to conventional design methods, including accepted codes and standards, is not sufficient to prevent these structural damage. No wind loading standard or code, can claim to cover complete wind loading. The present research was on the study of circular steel silo of medium height subjected to rigorous wind loading and its effects on the anchorage requirements. The conventional methods of analysis give more conservative results and hence extra cost to the client. This paper reports the comparison of results by various conventional and random analysis of medium silos for additional bending moment and shear force in wind environment. The maximum bending moment values (Nm E+06) for silos with H/D ratio of 1.25 in static analysis, quasistatic (IS code), quasistatic (AS code), dynamic and random analysis techniques were found to be 8.93, 11.3, 24.3, 22.3 and 13.3 respectively and 13.3 is the exact value. The present analysis is on the basis of wind speed data around Delhi, but the model can be used for any wind speed and location with minor changes.

**Key words :** Wind excitation, Shear force, Bending moment, Probability density function, Aspect ratio.

Slender, flexible and lightweight structures are the results of innovation in materials and computerization. Rising cost of land also compel sky scappers construction in the whole world. Such structures generally exhibit an increased susceptibility to the action of winds. Wind causes lateral load on the structure. Wind loads intensity is random in nature and vary around the world. It is necessary to examine more than one wind load case to arrive on the exact solution. Therefore, while designing these structures, it has to be ensured that the performance of structures subjected to action of wind will be adequate at the lowest cost during their anticipated life from viewpoint of both structural safety and serviceability. Effects of wind on structures are forced vibrations and self excited vibrations and their magnitude depends upon area of the exposure and the shape of the structure and many other factors. The conventional procedure of design and analysis includes the assumption of wind load as static or quasi-static. In peak wind approach, depending upon parameters such as risk level,

local topography, terrain roughness, height and size of the silo, design wind pressure is determined. From value of design wind pressure, wind load is determined using pressure coefficients and frontal area of the silo. For using mean wind approach, besides the parameters required for peak wind approach (1), value of Gust Factor is required for which fundamental natural frequency of the structure is required to be estimated. This was not applicable for highly flexible thin walled structures subjected to no internal pressure, e.g. silos during empty condition and unfavorable climatic conditions. The wind load was taken as uniformly distributed or considered as triangular linear variation or even trapezoidal. This concept was not a valid one for thin walled structures such as surface type silo especially made of thin steel walls. This called for the spatial behavior of the slender structure in the vicinity of other structures by a random wind loading. The anchorage requirement for the empty silo considering wind load effects was analyzed (2). They adopted the quasi-static wind load approach. This

had got limitations when the wind load had a random nature and the aerodynamic characteristics of structure were not considered.

The wind load effect through a mathematical model was analyzed approximately to include the equivalent wind spectrum technique considering the schematization of the wind as a stochastic stationary Gaussian process (3). Wind loads intensity is random in nature, hence stochastic approach was used. The high-frequency base technique is applicable to the large majority of tall structures and has the advantage of allowing the structural engineer to easily update the results in the case of a change in the structure's stiffness, mass or natural frequency. Several computer programs were prepared by the author for using this technique.

### Methods

The intermediate silos investigated had height to diameter (H/D) ratio of 1.25, 1.43, 1.67, 2.0, 2.5 and 3.0 with a single height of 15 m silo subjected to the condition of height to diameter ratio is less than 3.0 but more than 1.0. This intermediate silos choice was based on the study reported by Briassoulis and Packnold (2) and selected as this ratio represented a workable H/D ratio (aspect ratio) in this region. The concept of dynamic grain pressure during the filling and emptying process was also based on the well established Janssen (4) theory for calculating the wall thickness in preliminary design. The vertical pressure  $q$  and horizontal pressure  $p$  at a depth 'y' are given by the equations;

$$q = (\gamma R/k \mu') (1 - e^{-k\mu' Y/R})$$

$$p = kq$$

where  $R$  is hydraulic radius (area of cross-section divided by the perimeter) of the silo. The buckling critical stress (5) in the wall was estimated for axial compression without hoop tension by

$$f_{cr} = (C_0 E t)/r$$

where

$$C_0 = 0.374/(\text{SQRT}(1.0 + (r/100t))) \text{ for } r/t, \leq 212$$

$$C_0 = 0.315/(\text{SQRT}(0.1 + (r/100t))) \text{ for } r/t > 212.$$

If  $f_{cr}$  computed from above exceeds  $3/8 f_y$ , then

**Table 1.** Required sheet thickness (mm) at various depth from top.

Diameter depth (m)	12.0m	10.5m	9.0m	7.5m	6.0m
0—5	5.2	4.5	3.8	3.2	2.5
6—10	6.8	5.8	4.9	4.0	3.3
11—15	7.9	6.8	5.6	4.7	4.0

$$f_{cr} = f_y (1.0 - 0.347 (f_y/f_{cro})^{0.6})$$

where  $f_{cro}$  is  $f_{cr}$  calculated from above equation.

Assuming  $f_y = 420 \text{ N/mm}^2$ , a check on the adequacy of wall thickness was applied taking advantage of the hoop tension using equations :

The axial compression with hoop tension as give by :

$$f_{ch} = (C_h E t)/r \leq 3/8 f_y$$

$$C_h = C_0 + (0.45 - C_0) (\rho / (\rho + 0.007))$$

where

$$\rho = (p/E) (r/t)^{1.5}$$

in which  $p$  is the internal lateral pressure in the bin. For design, allowable stress = critical stress/1.5 > maximum of hoop or meridional stress. The design thickness adopted are shown in Table 1. The wall thickness was not only going to be parameters to counter the wind load effects but the proper anchorage at the base against overturning of the top about the base. The present work was based particularly on the above background. For the purpose of computations, the steel of the cylindrical shell has a modulus of elasticity  $E=200 \text{ Gpa}$ , poisson's ratio of 0.30, and mass density of  $7,900 \text{ kg/m}^3$ . The approaches for the analysis of such structures under static, quasi-static were even dynamic (wind totally dynamic) response needed the thorough modification to include the random wind loading. The random wind loading could be accounted by a spectral loading with probability analysis to find the exact response. But then when the shell was empty and the wind load was extensive what was going to happen to the wall and at the base. Wind loading will be effective and severe when it will be empty, so the wind load under the circumstances was considered to be critical. In view of this the spec-

**Table 2.** Values of maximum base BM (Nm E + 06) for intermediate silos by various analysis.

Height	Dia	H/D	Static	Qstat-I	Qstat-A	Dynamic	Random
15.00	12.00	1.25	8.93E+00	1.13E+01	2.43E+01	2.23E+01	1.33E+01
15.00	10.50	1.43	7.82E+00	9.93E+00	2.14E+01	1.95E+01	1.16E+01
15.00	9.00	1.67	6.70E+00	8.52E+00	1.82E+01	1.68E+01	1.01E+01
15.00	7.50	2.00	5.58E+00	7.07E+00	1.51E+01	1.39E+01	8.25E+00
15.00	6.00	2.50	4.47E+00	5.67E+00	1.20E+01	1.12E+01	6.37E+00
15.00	5.00	3.00	3.72E+00	4.72E+00	9.96E+00	9.30E+00	5.23E+00

tral analysis technique, which considered the response of random nature, was of value in this study. The study of wind effect was paramount important especially for the proper anchorage requirements of designed to stabilize the entire structure against overturning. The entire structure was assumed to be mathematically lumped at three places namely lower, middle, and top. After inverting the stiffness matrix the flexibility matrix was formed for conjugate beam method. Such analysis was quite common in the stress analysis of structures. With the use of the above matrix, first three natural frequencies were obtained. For random analysis, the wind velocity was divided into two parts namely, static part and fluctuating part. For the fluctuating part the basic mathematical wind model developed by Sharma (6), was followed for spectral analysis in this study. Relation between reduced frequency and energy was given as

$$X = \log(nz/V)$$

$$Y = \log n(S_n)$$

$$Y = -1.991225 - 0.4447708X - 0.1091633X^2$$

Relation between reduced frequency and normalized energy

$$X = \log(nz/V)$$

$$Y_1 = \log n(S_n) \sigma^2$$

$$Y_1 = -1.531011 - 0.2919729X - 0.1144482X^2$$

The mean square value of the fluctuating component of the response was calculated (7) following the standard procedure given in AS : 1170 (Part-2) 1989 as given below.

$$x^2 = 4(F^2/V^2k^2) \int_0^\infty \chi^2(n\sqrt{A}/V) |H(n)|^2 u^2 S_u(n) dn$$

Where F/K is the static displacement,  $\chi^2(n)$  = aerodynamic admittance function,  $|H(n)|^2$  = mechanical admittance,  $n$  = forcing frequency,  $u^2$  = Mean square fluctuating velocity,  $S_u(n)$  = power spectral density of fluctuating velocity,  $\sqrt{A}$  = characteristic dimension.

For probability analysis, extreme value type-1 distribution, probability distribution function curve were plotted (Fig 1, 2) for base BM and base SF. The brief procedure given by Benjamin and Allen (8) is given below :

$$\alpha = 1.282/\sigma_y$$

where  $\sigma_y$  = standard deviation,  $\alpha$  = measure of dispersion

$$u = m - 0.577/\alpha$$

where  $m$  = mean value,  $u$  = mode of the distribution

$$y = u + w/\alpha$$

where  $y$  is the variable for which PDF is to be plotted. Corresponding to arbitrary values of  $w$ , values of  $Fw(w)$  are available in the standard tables.

$$\text{Then } Fy(y) = \alpha \cdot Fw(w)$$

and  $y$  can found from above corresponding to every  $w$ .

Hence we can plot a curve between  $y$  and  $Fy(y)$  which is the required probability distribution function curve for variable  $y$ . For the distribution of largest values the mean value factor of maximum is given by

$$\eta_{(max)} = \sqrt{(2 \log_e vT) + 0.577/(\sqrt{(2 \log_e vT)})}$$

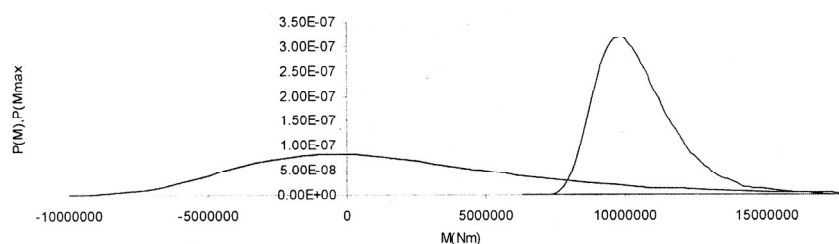


Figure 1. PDF of maximum bending moment at the base for silo height 15m.

and the standard deviation factor is given by

$$\sigma_{(\eta_{\max})} = (\pi/\sqrt{6}) (1/(\sqrt{2 \log_e vT}))$$

where  $v$ , the crossing rate is given by

$$= \sqrt{(\int_0^{\infty} n^2 S(n) dn / \int_0^{\infty} S(n) dn)}$$

The static part of the response due to the mean component of wind loading was calculated following IS : 875 (Part-3) 1987 (9). Mathematically design wind speed  $V_z$  can be expressed as

$$V_z = V_b K_1 K_2 K_3$$

where,  $V_z$  = design wind speed at any height  $z$  in m/sec.,  $V_b$ ,  $K_1$ ,  $K_2$ ,  $K_3$  as defined earlier.

*Design Wind Pressure.* The design wind pressure at any height above mean ground level was obtained by the following relationship between wind pressure and wind velocity :

$$P_z = 0.6V_z^2$$

where  $P_z$  = design wind pressure in Newton/m<sup>2</sup> at height  $z$ , and  $V_z$  = design wind velocity in m/sec at height  $z$ .

In regard to the consideration of a time series effect, one-hour interval time was adopted as proposed by Davenport (10). For probability analysis extreme value type-1 distribution was used and the computer program was developed accordingly.

## Results and Discussion

Bending moment and shear force at the base of each silo were calculated. The probability analysis

suggested by Davenport (10) was used to obtain the maximum probable values. The design values of bending moment and shear force at the base of each silo have been presented in Tables 2 and 3.

### Comparison of Bending Moment for Intermediate Silos

Results of the maximum bending moment for intermediate silos (Table 2) show that corresponding to the maximum bending moment at the base of silo by the static analysis, the maximum bending moment by quasi-static analysis using Indian code, quasi-static analysis using Australian code, dynamic analysis and random analysis are 26.54, 172.11, 149.72 and 48.94% higher for 15m high and 12m diameter, 26.98, 173.66, 149.36 and 48.34% higher for 15m high and 10.5m diameter, 27.16, 171.64, 150.74 and 50.75% higher for 15m high and 9m diameter, 26.70, 170.61, 149.10 and 47.85% higher for 15m high and 7.5m diameter, 26.85, 168.46, 150.56 and 42.50% higher for 15m high and 6m diameter and 26.88, 167.74, 150.00 and 40.59% higher for 15m high and 5m diameter.

In static analysis the maximum bending moment observed at the base for silo having 15m height and 10.5m diameter was 12.42% lower than the maximum bending moment at the base for silo having 15m height and 12m diameter. It was 14.32% lower for silo having 15m height and 9m diameter than for silo having 15m height and 10.5m diameter, 16.72% lower for silo having 15m height and 7.5m diameter than for silo having 15m height and 9m diameter, 19.89% lower for silo having 15m height and 6m diameter than for silo having 15m height and 7.5m diameter, 16.78% lower for silo having 15m height and 4.4m diameter than for silo having 15m height and 6m diameter.

In quasi-static analysis using Indian code the

**Table 3.** Values of maximum base SF (N) for intermediate silos by various analysis.

Height	Dia	H/D	Static	Qstat-1	Qstat-A	Dynamic	Random
15.00	12.00	1.25	1.18E+06	1.50E+06	3.22E+06	2.95E+06	1.76E+06
15.00	10.50	1.43	1.03E+06	1.31E+06	2.83E+06	2.58E+06	1.53E+06
15.00	9.00	1.67	8.86E+05	1.13E+06	2.41E+06	2.23E+06	1.33E+06
15.00	7.50	2.00	7.39E+05	9.35E+05	2.00E+06	1.84E+06	1.09E+06
15.00	6.00	2.50	5.91E+05	7.50E+05	1.59E+06	1.48E+06	8.43E+05
15.00	5.00	3.00	4.93E+05	6.25E+05	1.32E+06	1.23E+06	6.92E+05

maximum bending moment observed at the base for silo having 15m height and 10.5m diameter was 12.12% lower than the maximum bending moment at the base for silo having 15m height and 12m diameter. It was 14.20% lower for silo having 15m height and 9m diameter than for silo having 15m height and 10.5m diameter, 17.02% lower for silo having 15m height and 7.5m diameter than for silo having 15m height and 9m diameter, 19.80% lower for silo having 15m height and 6m than for silo having 15m height and 7.5m diameter, 18.34 percent lower for silo having 15m height and 4.4m than for silo having 15m height and 6m diameter.

In quasi-static analysis using Australian code the maximum bending moment observed at the base for silo having 15m height and 10.5m diameter was 11.93% lower than the maximum bending moment at the base for silo having 15m height and 12m diameter. It was 14.95% lower for silo having 15m height and 9m diameter than for silo having 15m height and 10.5m diameter, 17.03% lower for having 15m height and 7.5m diameter than for silo having 15m height and 9m diameter, 20.53% lower for silo having 15m height and 6m diameter than for silo having 15m height and 7.5m diameter, 17.00% lower for silo having 15m height and 4.4m diameter for silo having 15m height and 6m diameter.

In dynamic analysis the maximum bending moment observed at the base for silo having 15m height and 10.5m diameter was 12.56% lower than the maximum bending moment at the base for having 15m height and 12m diameter. It was 13.85% lower observed for silo having 15m height and 9m diameter than for silo having 15m height and 10.5m diameter, 17.26% lower for silo having 15m height and 7.5m diameter than for silo having 15m height and 9m diameter, 19.42% lower for silo having 15m height and 6m diameter than for silo having 15m height and 7.5m

diameter, 16.96% lower for silo having 15m height and 4.4m diameter than for silo having 15m height and 6m diameter. In random analysis the maximum bending moment at the base observed for silo having 15m height and 10.5m diameter was 12.78% lower than the maximum bending moment at the base observed for silo having 15m height and 12m diameter.

In random analysis the maximum bending moment observed at the base for silo having 15m height and 9m diameter was 12.93% lower than the maximum bending moment at the base for silo having 15m height and 10.5m diameter. It was 18.32% lower for silo having 15m height and 7.5m diameter than for silo having 15m height and 9m diameter, 22.79% lower for silo having 15m height and 6m diameter than for silo having 15m height and 7.5m diameter and was 17.90% lower for silo having 15m height and 4.4m diameter than for silo having 15m height and 6m diameter.

#### *Comparison of Shear Force for Intermediate Silos*

Table 3 shows that corresponding to the maximum shear force at the base by the static analysis, the maximum shear force at the base by quasi-static analysis using Indian code, quasi-static analysis using Australian code, dynamic analysis and random analysis are 27.12, 172.88, 150.00 and 49.15% higher than the silo for 15m high and 12m diameter, 27.18, 174.75, 150.48 and 48.54% higher for 15m high and 10.5m diameter, 27.54, 172.00, 151.69 and 50.11% higher for 15m high and 9m diameter, 26.52, 170.64, 148.98 and 47.50% higher for 15m high and 7.5m diameter, 26.90, 169.03, 150.42 and 42.64 percent higher for 15m high and 6m diameter and 26.77, 167.75, 149.49 and 40.36% higher for 15m high and 5m diameter.

In static analysis the maximum shear force observed at the base for silo having 15m height and

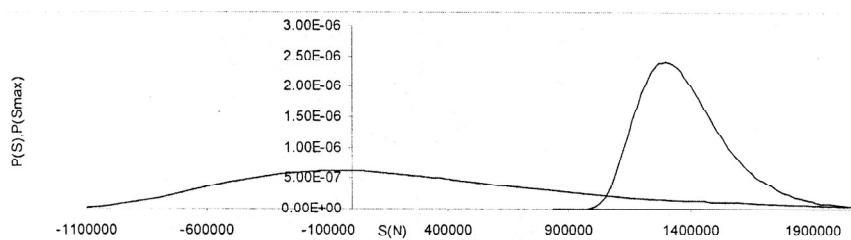


Figure 2. PDF of maximum shear force at the base for silo height 15 m.

10.5m diameter was 12.71% lower than the maximum shear force at the base for silo having 15m height and 12m diameter. It was 13.98% lower for silo having 15m height and 9m diameter than for silo having 15m height and 10.5m diameter, 16.59% lower for silo having 15m height and 7.5m diameter than for silo having 15m height and 9m diameter, 20.02% lower for silo having 15m height and 6m diameter than for silo having 15m height and 7.5m diameter and 18.88% lower for silo having 15m height and 4.4m diameter than for silo having 15m height and 6m diameter.

In quasi-static analysis using Indian code the maximum shear force observed at the base for silo having 15m height and 10.5m diameter was 12.67% lower than the maximum shear force at the base for silo having 15m height and 12m diameter. It was 13.74% lower for silo having 15m height and 9m diameter than for silo having 15m height and 10.5m diameter, 17.26% lower for silo having 15m height and 7.5m diameter than for silo having 15m height and 9m diameter, 19.79% lower for silo having 15m height and 6m diameter than for silo having 15m height and 7.5m diameter and 16.67% lower for silo having 15m height and 4.4m diameter than for silo having 15m height and 6m diameter.

In quasi-static analysis using Australian code the maximum shear force observed at the base for silo having 15m height and 10.5m diameter was 12.11% lower than the maximum shear force at the base for silo having 15m height and 12m diameter. It was 14.48% lower for silo having 15m height and 9m diameter than for silo having 15m height and 10.5m diameter, 17.01% lower for silo having 15m height and 7.5m diameter than for silo having 15m height and 9m diameter, 20.50% lower for silo having 15m height and 6m diameter than for silo having 15m height and 7.5m diameter and 16.98% lower for silo having 15 m height and

4.4 m diameter than for silo having 15m height and 6m diameter.

In dynamic analysis the maximum shear force observed at the base for silo having 15m height and 10.5m diameter was 12.54% lower than the maximum shear force at the base for silo having 15m height and 12m diameter. It was 13.65% lower for silo having 15m height and 9m diameter than for silo having 15m height and 1.5m diameter, 17.49% lower for silo having 15m height and 7.5m diameter than for silo having 15m height and 9m diameter, 19.56% lower for silo having 15m height and 6m diameter than for silo having 15m height and 7.5m diameter and was 16.89% lower for silo having 15m height and 4.4m diameter than for silo having 15m height and 6m diameter.

In random analysis the maximum shear force observed at the base for silo having 15m height and 10.5m diameter was 13.07% lower than the maximum shear force at the base for silo having 15m height and 12m diameter. It was 13.07% lower for silo having 15m height and 9m diameter than for silo having 15m height and 10.5m diameter, 18.04% lower for silo having 15m height and 7.5m diameter than for silo having 15m height and 9m diameter 22.66 percent lower for silo having 15m height and 6m diameter than for silo having 15m height and 7.5m diameter and was 17.91% lower for silo having 15m height and 4.4m diameter than for silo having 15m height and 6m diameter.

### Conclusions

The intermediate silos investigated had H/D ratio of 1.25, 1.43, 1.67, 2.0, 2.5 and 3.0. The maximum bending moment values (Nm E+06) for silos with H/D ratio of 1.25 in static analysis, quasistatic (IS code), quasistatic (AS code), dynamic and random analysis techniques were found to be 8.93, 11.3, 24.3, 22.3 and

13.3 respectively. The values of maximum bending moment at the base for other H/D ratios exhibited similar trends. The maximum shear force values (N E+05) for silos with H/D ratio of 1.25 of silos in static analysis, quasi-static (IS code), quasi-static (AS code), dynamic and random analysis techniques were found to be 11.8, 15.0, 32.2, 29.5 and 17.6 respectively. The values of maximum shear force at the base for other H/D ratios exhibited similar trends. For intermediate silos height, there was gradual decrease in maximum values of B.M and S.F. with the increase in height to diameter ratio from 1.25 to 3.0. In the present investigation following conclusion are drawn : The response of a silo structure to the random wind forces can be represented fairly accurately considering only the first two modes of vibrations. The response contribution from fluctuating component of wind is not high for silos of 15 meter height with H/D ratio less than 3.0. The gust factor approach is recommended for this category of silos. The response contribution from fluctuating component of wind is mild for 15m high silos with H/D ratio between 1.0 to 3.0. Therefore random analysis approach may be used for this category of silos being in the transition phase to avoid serious mistake and extra cost.

### References

1. Singh J., M. L. Bansal and V. R. Sharma. 2008. Peak and mean wind approach-A case study of silos. *Proc. Int. Conf. INSHAB CIT, Coimbatore, India.*
2. Briassoulisa D. and D. A. Packnold. 1986. Anchorage requirements for wind loaded empty silos. *J. Struct. Engg. ASCE* 112 : 308—324.
3. Solari. 1988. Equivalent wind spectrum technique : Theory and applications. *J. Struct. Engg. ASCE* 114 : 1303—1323.
4. Janssen H. A. 1895. On the pressure of grain in silos. *Inst. Civil Engg. London* 124 : 553—555.
5. Rajgopalan K. 1989. *Storage structures*. Oxford & IBH Publ. Co., New Delhi, India. 263—310 pp.
6. Sharma V. R. 1993. *Spectral characteristics and extreme value analysis for winds in India and codal provisions*. Ph.D. thesis. Volumes I & II, IIT, New Delhi, India.
7. AS 1170.2. 1989. *SAA loading Code*. Part 2. Wind loads. Standard Assoc. Australia, Australia.
8. Benjamin J. R. and C. Allen. 1975. *Probability, statistics and decisions for civil engineers*. McGraw Hill, New York, USA. 271—285 pp.
9. IS : 875 (part-3). 1987. *Indian standard code of practice for design loads (other than earthquake for buildings and structures)*. New Delhi, India.
10. Davenport A. G. 1964. Note on the distribution of the largest value of a random function with application to gust loading. Pp. 187—196. *Proc. 28th, Inst. of Civil Engs.* London, UK.