

Identifying Best Probability Distribution for Events of Maximum Rainfall—A Case Study of Karnal, Haryana

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Abstract The present study has been undertaken to fit best probability distribution of rainfall of Karnal District representing eastern climatic zone of Haryana state. For the purpose, daily rainfall data of 47 years was analyzed and processed to identify the maximum rainfall received on daily, monthly, seasonal and annual basis. The analysis showed that the maximum daily rainfall among the years ranged between 38 mm (1980) to 229 mm (2009) indicating a very large variation during the period of study. The standard deviation and coefficient of variation were also computed. Different goodness of fit tests were carried out to select the best fit probability distribution on the basis of highest rank with minimum value of test statistic. The probability distribution was identified using maximum overall score based on sum of individual point score obtained from different selected good-

ness of fit tests. Generalized extreme value distribution was found to be best fit probability distribution for most of the periods. The results clearly established that the analytical procedure devised and tested in this study may be suitably applied for the identification of the best fit probability distribution of weather parameters.

Keywords Maximum rainfall, Probability distributions, Goodness-of-fit tests, Pearson type distribution, Log-normal.

Introduction

A good understanding of the pattern and distribution of rainfall is of vital importance for water resource management of a country. Analysis of rainfall data strongly depends on its distribution pattern over time and it has great influence on agricultural production. It has long been a topic of interest in the fields of climatology to find a probability distribution that provides a good fit to daily rainfall. Several studies have been conducted in India and abroad on rainfall analysis and best fit probability distribution function such as Normal, Log-normal, gumbel, weibull and Pearson type distribution were identified.

Salami [1] studied the meteorological data for Texas and found that Gumbel distribution fits adequately for both evaporation and temperature data, while for precipitation data log-Pearson type III distribution confirms to be more accurate. Takara et al. [2] analyzed the extreme events and revealed that

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hydrological extremes sometimes do not fit well to the theoretical extreme-value distribution such as the Bumbel and generalized extreme value distributions. Lee [3] indicated that log-Pearson type III distribution fits for 50% of total station number for the rainfall distribution characteristics of Chia-Nan plain area.

Bhaskar et al. [4] observed the frequency analysis of consecutive days peaked rainfall at Banswara, Rajasthan, India, and found gamma distribution as the best fit distribution. Kwaku et al. [5] revealed that the log-normal distribution was the best fit probability distribution for one to five consecutive days' maximum rainfall for Accra, Ghana. Olofintoye et al. [6] examined that 50% of the total station number in Nigeria follows log-Pearson type III distribution for peak daily rainfall, while 40% and 10% of the total station follows Pearson type III and log-Gumbel distribution, respectively.

The present study was planned to establish the methodology for identifying the best fit probability distribution on the basis of three types goodness of fit tests for eastern agro-climatic zone of the state where water requirements are high owing to cultivation or rice crop. The Karnal district which represents the agro climatic properties of eastern agro-climatic zone was chosen for the purpose. The maximum rainfall data was used to select a best fit probability distribution for rainfall. The geographic location of Karnal district lies on the western bank of the river Yamuna, which forms its eastern boundary and separates Haryana from Uttar Pradesh. Karnal District lies between $29^{\circ}25'$ and 30° North and $76^{\circ}30'$ and $77^{\circ}10'$ East. Its height above mean sea level is around 253 meters [7].

Materials and Methods

The present study is based on time series data of maximums among daily rainfall in a year, season, month and week. The maximum daily, weekly, monthly, seasonal and annual rainfall data of 47 years (1966 to 2013) were collected from observatory of Central Soil Salinity Research Institute, Karnal. The normal an-

nual rainfall of the district is 582 mm with 32 rainy days in a year. About 82.39% of the annual rainfall is recorded during the southwest monsoon from July to September. August is the wettest month of the year with an average of 9.0 rainy days and 221.5 mm rainfall. Annual maximum rainfall of 1404 mm and minimum of 255 mm were observed in the years 1998 and 1987 respectively. The daily rainfall data were processed to identify the maximum rainfall received on any one day (24 h duration), in any week (7 days), in a month (4 weeks), in a season (monsoon season i.e. June to September as 82% of annual rainfall received in this season) and in a year (365 days period). Various probability distributions namely normal, lognormal (2P, 3P), gamma (2P, 3P), generalized gamma (3P, 4P), log-gamma, weibull (2P, 3P), Pearson 5 (2P, 3P), Pearson 6 (3P, 4P), log-Pearson 3, generalized extreme value were fitted and evaluated by using the following systematic steps.

Step I : Fitting the probability distribution

The description of various probability distribution functions viz. density function, range and the parameters involved are presented in Table 1.

Step II : Testing the goodness of fit

The goodness of fit tests viz. Kolmogorov–Smirnov test and Anderson–Darling test were used along with the chi-square test at α (0.01) level of significance for the selection of the best fit probability distribution.

Step III : Identification of best fit probability distribution.

Results and Discussion

The methodology presented above was applied to the 47 years weather data in which maximum rainfall (mm) events were taken from daily rainfall. The maximum daily rainfall ranged from 38 to 229 mm during the study period. The data was classified into 23 data sets as 1 annual (January to December), 1 seasonal (June to September), 4 months of rainy season (i.e. June, July, August and September) and 17 weeks (from Standard Meteorological Week no. 23 to 39) to

Table 1. Summary of statics for maximum daily rainfall.

Study period		Mean	SD	Skewness	CV	Max	Min	Quar-tile Q1	Quar-tile Q3
Annual	1 Jan-31 Dec	99.248	40.288	1.09	0.40594	229	38	69	120
Seasonal	1 Jun-30 Sep	97.781	40.457	1.551	0.41376	229	38	68.25	
Jun	1 Jun-30 Jun	37.948	22.444	0.59443	0.59143	98	12	22.4	48.45
Jul	1 Jul-31 Jul	71.942	50.052	1.3091	0.69574	229	5.8	36.35	99.35
Aug	1 Aug-31 Aug	65.692	37.351	0.60473	0.56858	158	0	35.1	94.075
Sep	1 Sep-30 Sep	46.34	33.701	0.5141	0.72726	124.4	0	15.375	67.1
1 week	4 Jun-10 Jun	8.2458	16.747	3.1424	2.031	88.2	0	0	11.05
2 week	11 Jun-17 Jun	14.006	19.949	2.126	1.4243	98	0	0	22.475
3 week	18 Jun-24 Jun	13.135	17.814	1.4502	1.3562	70	0	0	25.15
4 week	25 Jun-1 Jul	21.26	22.477	1.15	1.0572	79	0	3.2	36.55
5 week	2 Jul-8 Jul	25.344	38.862	2.2794	1.5334	165.8	0	0.3	32.8
6 week	9 Jul-15 Jul	37.8	43.878	2.1745	1.1608	229	0	4.1	5.2
7 week	16 Jul-22 Jul	30.027	37.233	2.4854	1.24	100.43	0	2.35	37.9
8 week	23 Jul-29 Jul	30.577	36.373	2.1088	1.1895	159.8	0	5.55	43.7
9 week	30 Jul-5 Aug	34.663	36.004	1.2176	1.0387	140.2	0	7.55	46.65
10 week	6 Aug-12 Aug	26.833	29.634	2.1596	1.1044	152	0	8.3	33.7
11 week	13 Aug-19 Aug	30.398	32.921	1.8012	1.083	158	0	6.825	45.25
12 week	20 Aug-26 Aug	26.33	33.543	1.6766	1.2738	124.8	0	1.7	35.75
13 week	27 Aug-2 Sep	22.521	27.938	1.493	1.2405	121	0	0	39.8
14 week	3 Sep-9 Sep	23.165	31.474	1.4663	1.3587	103.56	0	0	40.7
15 week	10 Sep-16 Sep	21.148	29.195	1.6602	1.3805	120.2	0	0	34.55
16 week	17 Sep-23 Sep	6.6583	13.883	2.6717	2.085	57.6	0	0	4.95
17 week	24 Sep-30 Sep	8.2896	22.834	3.6886	2.7546	124.4	0	0	2.2

study the distribution pattern at different levels. The summary statistics (mean, standard deviation, skewness coefficient, coefficient of variation, maximum and minimum values of daily maximum rainfall) are

presented in Table 1. It is observed that mean of maximum daily rainfall of all years is 99.3 mm, seasonal mean value is 97.8 mm. The seasons of monthly and weekly values ranged from 37.9—71.9 mm and 8.2—

Table 2. Study period wise first ranked probability distribution using goodness of fit tests.

Study period		Kolmogorov Smirnov		Anderson Darling			Statistics
		Distribution	Statistics	Distribution	Statistics	Chi-Square Distribution	
Annual	1 Jan-31 Dec	Log normal	.06115	Log Pearson 3	.17337	Gen. Gamma	0.14786
Seasonal	1 Jun-30 Sep	Log normal	.06256	Log Pearson 3	0.20168	Gen. Gamma	0.4011
Jun	1 Jun-30 Jun	Gen. Extreme	.0741	Gen. Extreme	0.24015	Gamma	0.62061
Jul	1 Jul-31 Jul	Log Pearson 3	.05193	Log Pearson 3	0.17845	Pearson 6	0.55541
Aug	1 Aug-31 Aug	Log Pearson 3	.05424	Log Pearson 3	0.14333	Pearson 6	0.87318
Sep	1 Sep-30 Sep	Gen. Gamma (4P)	.07823	Gen. Extreme Value	0.54376	Gen. Gamma 4P	0.43922
1 week	4 Jun-10 Jun	Log normal	.31123	Pearson 6	-6.4889	Gen. Extreme Value	14.693
2 week	11 Jun-17 Jun	Gen. Extreme Value	0.22708	Gen. Extreme Value	2.6949	Normal	5.884
3 week	18 Jun-24 Jun	Gen. Extreme Value	0.22231	Gen. Extreme Value	2.9553	Gen. Extreme Value	6.4497
4 week	25 Jun-1 Jul	Gen. Extreme Value	0.1117	Gen. Extreme Value	.95025	Gen. Extreme Value	.47341
5 week	2 Jul-8 Jul	Gen. Extreme		Gen. Extreme		Gen. Extreme	

Table 2. Continued.

Study period		Kolmogorov	Smirnov	Anderson Darling	Statistics	Chi-Square	Statistics
		Distribution	Statistics	Distribution		Distribution	
6 week	9 Jul-15 Jul	Value	0.15695	Value	1.43	Value	2.8428
		Gen. Extreme		Gen. Extreme		Gen. Extreme	
7 week	16 Jul-22 Jul	Value	0.11291	Value	.7145	Value	3.5071
		Gen. Extreme		Gen. Extreme		Gen. Extreme	
8 week	23 Jul-29 Jul	Value	0.12178	Value	.75757	Value	3.609
		Gen. Gamma		Gen. Extreme		Gamma	
9 week	30 Jul-5 Aug	(4p)	0.0882	Value	.75503	Gamma	1.1012
		Gen. Extreme		Gen. Extreme		Gen. Gamma	
10 week	6 Aug-12 Aug	Value	0.10881	Value	0.83787	Gen. Gamma	2.159
		Gen. Extreme		Gen. Extreme		Gamma	
11 week	13 Aug-19 Aug	Value	0.07612	Value	.45907	Gamma	0.51205
		Gen. Extreme		Gen. Extreme		Weibull	
12 week	20 Aug-26 Aug	Value	0.9647	Value	0.55914	Weibull	0.49948
		Gen. Extreme		Gen. Extreme		Gen. Extreme	
13 week	27 Aug-2 Sep	Value	0.1359	Value	1.1737	Value	2.763
		Gen. Extreme		Gen. Extreme		Gen. Extreme	
14 week	3 Sep-9 Sep	Value	0.18118	Value	2.1432	Value	6.6409
		Gen. Extreme		Gen. Extreme		Gen. Extreme	
15 week	10 Sep-16 Sep	Value	0.16957	Value	2.3093	Value	5.8423
		Gen. Extreme		Gen. Extreme		Gen. Extreme	
16 week	17 Sep-23 Sep	Value	0.16827	Value	1.9813	Value	7.2851
		Gen. Extreme		Gen. Extreme		Gen. Extreme	
17 week	24 Sep-30 Sep	Value	0.28211	Weibull	-7.5802	Value	10.573
		Normal	0.37841	Weibull	-1.3423	Normal	23.265

37.8 mm, respectively. The maximum daily rainfall in a year / monsoon season was 229 mm during different months (i.e. June, July, August and September) of

monsoon period, the maximum daily rainfall ranged from 98—229 mm. The weekly maximum daily rainfall ranged from 70—229 mm. The minimum among the

Table 3. Parameters of the best fitted distribution.

Study period	Distribution	Parameters
Annual	1 Jan–31 Dec	Lognormal
	Log-Pearson 3	$\sigma = 0.38964$ $\mu = 4.5217$
	Gen. Gamma	$\alpha = 4203.0$ $\beta = 0.00607$ $\gamma = -21.006$
Seasonal	1 Jun–30 Sep	Lognormal
	Log-Pearson 3	$k = 1.0245$ $\alpha = 6.3612$ $\beta = 16.354$
	Gen. Gamma	$= 0.39196$ $= 4.5052$
Jun	1 Jun–30 Jun	Lognormal
	Log-Pearson 3	$= 337.79$ $= 0.02155$ $= -2.7749$
	Gen. Gamma	$k = 1.0286$ $= 6.1659$ $= 16.739$
Jul	1 Jul–31 Jul	Gen. Extreme Value
	Gamma	$k = -0.10392$ $\sigma = 19.852$ $\mu = 28.355$
	Log-Pearson 3	$\alpha = 2.8589$ $\beta = 13.274$
Aug	1 Aug–31 Aug	Log-Pearson 3
	Pearson 6	$\alpha = 95.897$ $\beta = -0.07235$ $\gamma = 10.983$
	Log-Pearson 3	$\alpha_1 = 3.1943$ $\alpha_2 = 8.0491$ $\beta = 159.19$
Sep	1 Sep–30 Sep	Pearson 6
	Gen. Gamma	$\alpha = 5.292$ $\beta = 0.29631$ $\gamma = 5.562$
	Gen. Extreme Value	$\alpha_1 = 2.7836$ $\alpha_2 = 8821.8$ $\beta = 2.0769E+5$
1 week	4 Jun–10 Jun	Gen. Gamma
	Weibull	$k = 4.0742$ $\alpha = 0.21883$ $\beta = 104.2$
	Gen. Extreme Value	$k = -0.08196$ $\sigma = 29.73$ $\mu = 31.423$
	Normal	$\sigma = 16.747$ $\mu = 8.2458$
	Weibull	$\alpha = 0.25448$ $\beta = 1.1713$
	Gen. Extreme Value	$k = 0.64078$ $\sigma = 3.0883$ $\mu = 1.121$

Table 3. Continued.

Study Period		Distribution	Parameters		
2 week	11 Jun–17 Jun	Gen. Extreme Value	$k = 0.40414$	$\sigma = 8.0118$	$\mu = 4.1184$
		Normal	$\sigma = 19.949$	$\mu = 14.006$	
3 week	18 Jun–24 Jun	Gen. Extreme Value	$k = 0.37997$	$\sigma = 7.8561$	$\mu = 3.9353$
4 week	25 Jun–1 Jul	Gen. Extreme Value	$k = 0.22053$	$\sigma = 13.511$	$\mu = 9.736$

Table 3. Continued.

Study period		Distribution	Parameters		
5 week	2 Jul–8 Jul	Gen. Extreme Value	$k = 0.48495$	$\sigma = 12.54$	$\mu = 6.6793$
6 week	9 Jul–15 Jul	Gen. Extreme Value	$k = 0.27124$	$\sigma = 22.709$	$\mu = 16.472$
7 week	16 Jul–22 Jul	Gen. Extreme Value	$k = 0.32116$	$\sigma = 17.168$	$\mu = 12.233$
8 week	23 Jul–29 Jul	Gen. Gamma	$k = 0.36909$	$\alpha = 5.1339$	$\beta = 0.25284$
		Gen. Extreme Value	$k = 0.36194$	$\sigma = 15.821$	$\mu = 12.745$
		Gamma	$\alpha = 0.70671$	$\beta = 43.267$	
9 week	30 Jul–5 Aug	Gen. Extreme Value	$k = 0.21816$	$\sigma = 21.641$	$\mu = 16.285$
		Gen. Gamma	$k = 0.9585$	$\alpha = 0.96901$	$\beta = 37.397$
10 week	6 Aug–12 Aug	Gen. Extreme Value	$k = 0.29221$	$\sigma = 14.657$	$\mu = 12.494$
		Gamma	$\alpha = 0.81989$	$\beta = 32.728$	
11 week	13 Aug–19 Aug	Gen. Extreme Value	$k = 0.24223$	$\sigma = 18.353$	$\mu = 14.092$
		Weibull	$\alpha = 0.24405$	$\beta = 21.643$	
12 week	20 Aug–26 Aug	Gen. Extreme Value	$k = 0.35764$	$\sigma = 15.227$	$\mu = 9.3235$
13 week	27 Aug–2 Sep	Gen. Extreme Value	$k = 0.30365$	$\sigma = 14.313$	$\mu = 8.1973$
14 week	3 Sep–9 Sep	Gen. Extreme Value	$k = 0.39198$	$\sigma = 13.504$	$\mu = 6.9359$
15 week	10 Sep–16 Sep	Gen. Extreme Value	$k = 0.40823$	$\sigma = 11.958$	$\mu = 6.2565$
16 week	17 Sep–23 Sep	Gen. Extreme Value	$k = 0.67349$	$\sigma = 2.2597$	$\mu = 0.82868$
		Weibull	$\alpha = 0.29197$	$\beta = 1.2107$	
17 week	24 Sep–30 Sep	Normal	$\sigma = 22.834$	$\mu = 8.2896$	
		Weibull	$\alpha = 0.28446$	$\beta = 0.94792$	

maximum daily rainfall was 38 mm for annual. The maximum value of coefficient of variation was observed in the seventeen week which indicates a large fluctuation in the rainfall data set and minimum value of coefficient of variation 0.406 was observed for the whole year which shows that fluctuation was minimum for the whole year. The test statistics D , A^2 and χ^2 for each data set were computed for sixteen probability distributions and the probability distribution having the first rank along with their test statistic is presented in Table 2. It was observed that Lognormal distribution using Kolmogorov Smirnov test, log-Pearson 3 value using Anderson Darling test and Gen. Gamma using Chi-square test obtained the first rank for maximum daily Annual rainfall as well as for seasonal rainfall. Thus the three probability distributions were identified as the best fit based on these

three tests independently. In Table 3 parameters of the distribution was identified.

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