

Hydroclimatic Variables with the Aid of Wavelet Transformation under Time Series Analysis

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Abstract Wavelet transformation is relatively new in pure and applied mathematics. They have a strong relation with Fourier analysis with respect of theory and applications. Wavelet analysis in hydroclimatic variables, decompose the original sample based time series data and then extracts wavelet scale information. Wavelet techniques are basically used for integrating the nucleus of the analysis to gather more information about the processes and basic characteristics behind the process. Many studies have been brought out with the help of wavelet analysis in the field of hydroclimatic variables on both international,

national level. Some selected papers are shortly described in this paper.

Keywords Hydroclimatic variables, Time series, Wavelet, Transformation.

Introduction

The climate changes have attracted wide interest in the fields of hydrology, meteorology and climatology. Climate is continuously changing over time in all over the world and the investigation of these changes is linked with trends in different indices of climate. In order to detect the existence of trends many studies at different time have been conducted in the fields of hydro-climatology and hydro-meteorology. Ecosystem, driven by the fluctuation of different hydroclimatic variables, at different time is often unpredictable from the usual standing point of living system of individuals. Now a days there is question or say, a grand challenge for the researcher to evaluate how the fluctuations of hydroclimatic variables at different time scales changes the frequency and intensity of extreme events and also the effect of changes on ecosystem. Most of the studies conducted to investigate the impacts of climate change on water resources concluded that the climate change have an adverse impact on water resources, both on quality and quantity. Trend analysis in hydroclimatic variables is challenging due to their non-stationary nature and the presence of noise and stochastic com-

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ponents in them. Non-stationary characteristics may occur in different time scale in the system of climate. By analyzing the various components of climatic and hydrologic time series such as, temperature, rainfall, evaporation steamflow, water quality all over the world, researchers can see the presence or absence of trend. Mann-Kandell and Modified Mann-Kandell are the most popular non-parametric methods for analyzing trend in hydroclimatic time series. Again, the wavelet transformation can be used to analyze non-stationary time series by transforming original data of time domain to time-frequency representation.

Hydrology is the scientific study of the movement, distribution and quality of water on earth and other planet. Climatology is the study of climate, more generally it studies the weather conditions averaged over a long period of time. Hydroclimatology is defined as the study of the influence of climate upon the water of the land. The parameters which are studied in context of hydroclimatology are known as hydroclimatic variables. Temperature, rainfall, steam flow, precipitation are some of those hydroclimatic variables. Hydroclimatology provides a systematic structure for analyzing how the climate system causes time and space variations in the hydrologic cycle. The changes in the climate condition and the hydrologic cycle underlie floods and possible future influences of global warming on water resources. Analysis on hydroclimatic variables can provide information on how the climate has evolved over time. This can be accomplished through time series analysis. Trend analysis in hydroclimatic variables is challenging due to their non-stationary nature and the presence of noise and stochastic components in them. Though surface air temperature has been accepted as the most obvious indicator of global climate change, surface precipitation, rainfall, are recognized as an equally important indicator of changing global climate (Hulme 1994). It is hardly possible to enumerate all the components of climate, much less describe and their contribution. Also it is necessary to take account the impact of external forces on climate. One of the approaches to deal with this situation is time series investigation, detecting cycle and trends and their variation in some time scale (Burroughs 1992). But, meteorological time series are non-stationary in nature and so sometimes it is necessary to apply some

advance transformation to decompose a series in time-frequency domain (Pisofat et al. 2004).

Time Series Analysis and Wavelet Transformation

In general, time series is the records of a process which may vary over time. Observations or recordings are either continuous or discrete in nature. Let (Ω, A, P) be a probability space and let T be an index set. A real valued time series is a real valued function $X(t, \omega)$, defined as $T \times \Omega$ such that for each fixed t , $X(t, \omega)$ is a random value on (Ω, A, P) . The function $X(t, \omega)$ is often written as $X_t(\omega)$ or X_t and time series can be considered as a collection $\{X_t, t \in T\}$ of random variable. For fix ω , $X_t(\omega)$ is a real valued function of t . This function is called a realization for a sample function. If the index set contains exactly one element then the stochastic process is a single random variable and we have to define the distribution function of the process. For stochastic process with more than one random variable we need to consider the joint distribution function-

$$F_{x_{t_1}, x_{t_2}, \dots, x_{t_n}}(x_{t_1}, x_{t_2}, \dots, x_{t_n}) = P\{\omega \mid x(t, \omega) \leq x_{t_1}, \dots, x(t_n, \omega) \leq x_{t_n}\}$$

Where, $\{x_{t_1}, x_{t_2}, \dots, x_{t_n}\}$ is a finite set of random numbers from the collection $\{X_t; t \in T\}$

In time series analysis, the most important things which are of interest are: (a) To describe the important features of the time series or modelling, (b) To explain how the past affects the future smoothing, (c) To forecast future values of the time series, (d) To assist as a control for a variable that measures the quality of product by the time series.

The classical decomposition is one of the simple methods of describing a time series. The four elements through which the series can be decomposed into are: Trend (T_t)—long term movements, Seasonal effects (S_t)—cyclical variations related to the calendar, Cycles (C_t)—other cyclical fluctuations, Random effects (E_t)—other random or systematic fluctuations.

The main focus is to create separate models with



Fig. 1. Wavelet transformation.

these four elements either additively

$$X_t = T_t + S_t + C_t + E_t$$

or multiplicatively,

$$X_t = T_t \cdot S_t \cdot C_t \cdot E_t$$

The mean and the variance of random variables have a special place in the theory of statistics. In time series analysis, the analogs of these are the mean function and the autocovariance function. Let us assume that, the time series values are the realizations of random variables Y_1, Y_2, \dots, Y_T , which are the part of a stochastic process $\{Y_t : t \in Z\}$. The mean function of a time series is defined to be

$$\mu(t) = E(Y_t)$$

and the autocovariance function is defined as

$$\gamma(s, t) = \text{cov}(Y_s, Y_t)$$

There are $2T + T(T - 1)/2$ parameters associated with the time series Y_1, Y_2, \dots, Y_T and hence it is quite tough to estimate all these parameters. So, for making any progress at all we must carry out constraints on the time series. The most common constraint is that of stationarity. Stationarity is a key idea for a time series. A time series is called stationary, if its behavior does not change over time, which means, the values always tend to vary about the same level and their variability is constant over time.

Strict Stationarity

A time series $\{Y_t : t \in Z\}$ is said to be strictly stationary if the distribution of $(Y_{t_1}, Y_{t_2}, \dots, Y_{t_k})$ is the same as that

for $(Y_{t_1+u}, Y_{t_2+u}, \dots, Y_{t_k+u})$ for every value of u ; where $k > 0$ and $t_1, t_2, \dots, t_k \in Z$.

Weak Stationarity

If $E[Y_t] = \mu, E[Y_t]^2 < \infty$ and $\gamma(t + k, t) = \gamma(k)$ for all t and u then a time series is said to be weakly stationary or covariance stationary or second-order stationary. The sequence $\{\gamma_k, k \in Z\}$ is called the autocovariance function and $\{\rho_k, k \in Z\}$ is the autocorrelation function (ACF) where

$$\rho_k = \gamma_k / \gamma_0 = \text{corr}(X_t, X_{t+k})$$

Time domain and frequency domain are two ways of looking at the same dynamic system. They are interchangeable. Interchangeable in the sense that, no information will lose in changing from one domain to another. They are complementary points of view that lead to a complete, clear understanding of the behavior of a dynamic system.

Time domain is the traditional way of observing signals. The time domain is a record of what happens to a parameter of the system in context of time. It is the analysis of mathematical functions, physical signals or time series of economic or environmental data, with respect to time. In the time domain, the signal or function's value is known for all real numbers, for the case of continuous time, or at various separate instants in the case of discrete time. The time domain graph shows how a signal changes with time, whereas a frequency domain graph shows how much of the signal lies within each given frequency band over a range of frequencies. Some of the time series approaches under time domain are- inspection of Autocorrelation, Partial Autocorrelation and Cross-

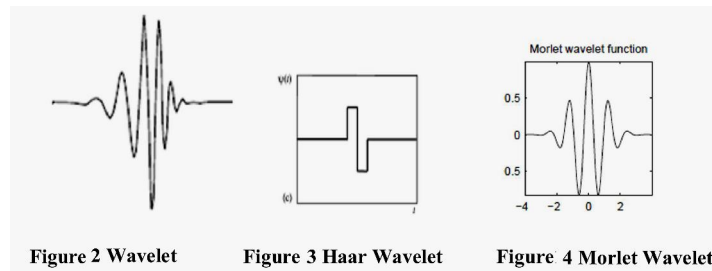


Fig. 2. Wavelet. Fig. 3. Haar wavelet. Fig. 4. Morlet wavelet.

correlation functions; modelling with AR, MA, ARMA, ARIMA, SARIMA, ARCH, GARCH, VAR. In the time domain statistical analysis the researchers have paid much attention to empirical risk minimization for last many years. Empirical risk minimization is rational as the empirical risk is approaching the expected risk when numbers of samples are large (Liu et al. 2012).

In electronics, control systems engineering, the frequency domain refers to the analysis of mathematical functions or signals with respect to frequency, rather than time. A time-domain graph shows how a signal changes over time whereas a frequency-domain graph shows how much of the signal lies within each given frequency band over a range of frequencies. A frequency-domain representation can also include information on the phase shift that must be applied to each sinusoid in order to be able to recombine the frequency components to recover the original time signal. A given function or signal can be converted between the time and frequency domains with a pair of mathematical operators called a transform. An example is the Fourier transform, which converts the time function into a sum of sine waves of different frequencies, each of which represents a frequency component. The inverse Fourier transform converts the frequency domain function back to a time function. There are a number of different mathematical transforms which are used to analyze time domain functions by transforming from time domain to frequency domain. These transformations are referred to frequency domain methods. The transformation such as, Fourier series, Fourier transform, Z trans-

form, Wavelet transform are referred as frequency domain methods. In the next section of this chapter, we have discussed upon some important studies on hydroclimatic variables undertaken at different levels (international and regional level) using both time domain and frequency domain approach.

Wavelet transformation is relatively new in mathematical sciences. They have a strong relation with Fourier analysis with respect to theory and applications. For the synthesis of ideas in the field of electrical engineering, physics, computer science, economics, wavelet transformation is an emerging procedure for the last twenty years. For its beautiful and deep mathematical property, wavelet analysis is capable of doing well in wide range functional spaces and different types of data (Chui 1992, Daubechies 1992, Mallat 1998). Wavelet analysis in hydroclimatic variables, decompose the original sample based time series data and then extracts wavelet scale information. This method is well known for its simplicity as well as it takes less time for calculation. For a time series, X_t , that has a continuous scale but a discrete recording sequence and $t = 0, \dots, t - 1$, then the wavelet function (Ψ), which depends on a time variable (n), is generally defined as (Partal and Küçük 2006):

$$\Psi(n) = \Psi(s, \gamma) = \frac{1}{\sqrt{s}} \Psi\left(\frac{t - \gamma}{s}\right)$$

Where t represents time; variable γ is the translation factor (time shift) of the wavelet over the time series; and variables ranging from 0 to $+\infty$ denotes the wave-

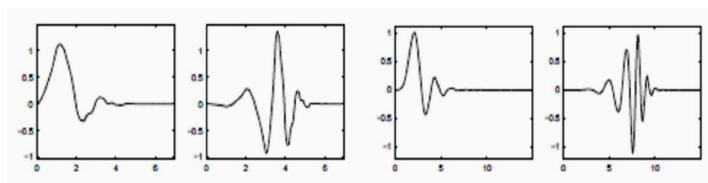


Fig. 5. Daubechies wavelet.

let scale (scale factor). DWT adopts the following form (Partal and Küçük 2006):

$$\Psi_{(a,b)} \left(\frac{t-\gamma}{s} \right) = \frac{1}{(s_0)^{a/2}} \Psi \left(\frac{t-b\gamma_0 s_0^a}{s_0^a} \right)$$

Ψ indicates the mother wavelet, a and b are integers, which represents the amount of expansion (scale factor) and translation of the wavelet respectively, s_0 symbolizes a dilation step whose value is unchanged and is greater than 1; and γ_0 denotes the location variable whose value is greater than zero.

A function to be a wavelet must satisfy the following conditions-wavelet must be centered at zero amplitude-

$$\int_{-\infty}^{\infty} \Psi(t) dt = 0$$

wavelet must have a finite energy-

$$\int_{-\infty}^{\infty} \Psi^2(t) dt < \infty$$

sufficient condition for inverse wavelet transforms is-

$$\int_{-\infty}^{\infty} \frac{\Psi(\omega)}{\omega} d\omega < \infty$$

Some of the popular wavelets are : (a) Haar Wavelet, (b) Morlet Wavelet, (c) Daubechies wavelet, (d) Meyer Wavelet, (e) Mexican Hat wavelet.

Wavelet Transformation in Hydroclimatic Studies

Wavelet strategies are essentially utilized for coordinating the core of the examination to accumulate more data about the procedures and fundamental qualities behind the procedure. Numerous investigations have been carried out with the assistance of wavelet analysis in the field of hydroclimatic variables on both global and national level. The wavelet change (WT) has pulled in much consideration since its theoretical development by Grossmann and Morlet (1984) with unique reference to geophysical seismic signal. Torrence and Compo (1998) looked at the Fourier analysis and WT. New test was produced to test the significance level of wavelet power spectra for white and red noise. Furthermore, it is demonstrated that, to expand the certainty in wavelet spectrum, smoothing in time and scale can be used. They utilized the recently created test on the information of Nino3 ocean surface temperature and the Southern Swaying and found an essentially higher power amid 1880-1920 and 1960-90, and lower control amid 1920-1960, and a conceivable 15 year variety. Amid the wet session, the turbulent collaborations among vertical breeze speed and temperature controlled in the Amazon had been examined by Bolzan and Vieira (2006). The strategy depends on the estimation of the correlation coefficient between the diverse scales in turbulent fields and Cross Wavelet Power. As indicated by the outcomes, the correlation among size of the vertical breeze speed are because of the intelligible structures, which implies, a substantial scale signature in the thermal profile. This intelligent structure characterizes an expansion in the communication among vertical breeze speed and temperature, and furthermore relies upon the atmospheric stability. Labat et al. (2001) in their investigation centered the

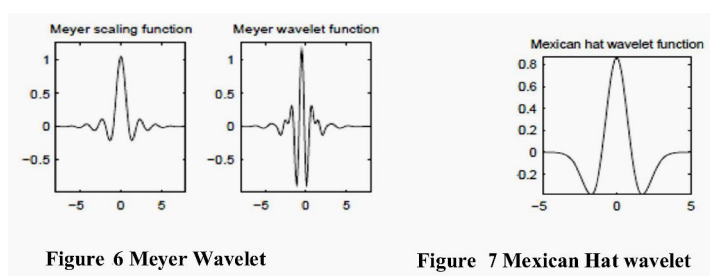


Fig. 6. Meyer wavelet. Fig. 7. Mexican hat wavelet.

utilization of wavelet strategy, especially the More WT in French karstic basin and contrasted their discoveries and traditional time arrangement examination like spectral and correlation analysis. A karstic development is a three dimensional hydrological basin. Once more, the continuous wavelet analysis put in confirm the multi scale conduct, for example, month to month, semiannual, yearly and so on in the framework. In prior examination, for non-stationary issue in karstic hydrology either non-linear Volterra models or other diverse models were utilized for temporal and physical case. In any case, in their examination the utilization of Morlet wavelet lessened the season of count and permits fast and productive decomposition of signals of segments without loss of data and permits better investigation of various segments as there is on need of time arrangement to be stationary. Barga and Santos (2010) found that, the wavelet power spectra had demonstrated a major power focus between the 8-16 month band of rainfall in Matsuyama city. By averaging all scales in the vicinity of 8 and 16 months, the time of low difference could be gotten. Wavelet analysis has been utilized to trace and evaluate variety in temporal pattern in various areas. Prokoph and Patterson (2004) had utilized regression and Wavelet transformation (WT) in atmosphere records of urban Ottawa and adjacent country territories in eastern Ontario. They had examined the conceivable connections between watched environmental change at the investigation territory and conceivable normal reason and watched that temperature in Ottawa expanded on the average 0.01°C every year in contrast with neighboring rustic territories of late circumstances. Likewise, the investigation demonstrated that this relative urban warming pat-

tern was fundamentally showed as multi-decadal and buries seasonal cycles and that were probably going to be identified with the progressive expanded in the temperature in the winter season in Ottawa which was related with the populace development. Santos et al. (2001) applied wavelet analysis on add up to month to month precipitation of Iberian Peninsula, Japan and north-eastern Brazil area to see the precipitation fluctuation and piece of precipitation of every region. The wavelet power spectra indicated enormous power focuses in the 8 to 16 month groups for every groups for every one of the locales. For Madrid and Barcelona, they additionally demonstrated a semi-yearly periodicity. Significant low frequency motions, identified with the North Atlantic Wavering, were recognized for the Iberian stations. Periods with low change in the 8 to 16-month band were recognized for Angicos, correspondent with one of the major droughty occasions in that semiarid piece of Brazil. WT uncovered imperative highlights of the rainfall time series. These outcomes empower its utilization, rather than the original series, in hydrological demonstrating contemplates. Kim (2004) said through his work, that the spatial pattern of the precipitation in Northern California, USA may have changed since 1945 and it ruled for a long time. Monotonic pattern that is assessed by wavelet change showed wetting in that area. The examination was done with the assistance of 96 years precipitation information. Kim additionally saw that, the pattern esteems utilizing ordinary least squares technique is deliberately bigger than that of wavelet analysis. This is because of the reason that the wavelet strategy isolates the diverse intermittent segments from the patterns, though the least squares technique joins the parts into one

aggregate pattern. Pisofat et al. (2004) broke down trend and oscillations in the mean monthly temperature time series in the Czech Republic. They found a positive pattern in temperature for each time arrangement set when they considered the distinction between unique time arrangement information and the information after reverse WT. They likewise got a thought of event of particular occasions. Amid the period 1930 to 2001, the wavelet power spectra indicated the centered based on 8 and 12-14 years for all arrangement. Like the connection of the temperature increment with anthropogenic-related variables, the expansion has likewise been related with air fluctuation. Yueqing et al. (2005), made an investigation in light of 45 years of rainfall information amid the period 1955-2000 of Hebei Plain of China utilizing Maxican Hat WT. They found there were clear intermittent swaying of 8-12 years and 4-6 years of seasonal and precipitation varieties. The trend in summer rainfall and yearly rainfall demonstrated fundamental times of 1 and 12 years. Earlier the Continuous Wavelet change (CWT) and the Mann-Kendall trend test was used for analyzing precipitation and stream flow in North-Western Oklahoma, USA during the period of 1894 to 2003. They outline how CWT analysis the investigation of information where anthropogenic effects were superimposed on the changeability of characteristic wonder, for example, EI Niño– Southern Oscillation and the Pacific Decadal Oscillation. By dissecting the temporal dispersion of pinnacles of the power spectra created by the CWT and correlating the information utilized with the huge teleconnection designs, they have done this work. Partal and Küçük (2006) break down the non-stationary attributes and in addition trend in Turkish precipitation information in three stations-Çanakkale, Balikersier, and Siirt stations utilizing WT technique. They connected CWT to acquire the data about the periodic structure of the information of the stations. After then the information were deteriorated into dyadic scales with the assistance of discrete wavelet change strategy. For the stations Çanakkale and Balikersier the occasions of the time scale 18-28 years were essential in creating trend. Once more, for the station Siirt, 4 year occasional mode, was mindful for trend. For recognizing breaking waves and non-breaking waves up the breeze wave time arrangement, some workers applied Morlet Wavelet through in his study.

The investigation demonstrated that the vacillation of the time arrangement of wind-wave are profoundly repetitive, which implies, the vitality at various scales varies observably with time. Partal (2010) utilized the discrete wavelet change (DWT) to consider drifts in Turkish stream flow series. The specialist drew out his examination on four stations-three of Sakarya Basin and one of Sehyan Basin. All the time arrangement from the four basin were disintegrated into four decomposition levels (2—16 years). The 16-year periodicity was observed to be most essential part that influences stream flow trend in Sakarya Basin. The 16-year segment had the most astounding correlation coefficient and the closest trend an incentive to the first information. Again the 8-year periodicity was observed to be in predominant mode in Sehyan Basin. This mirrors the prevailing intermittent segments which are in charge of pattern creation are diverse for various climatic locales in Turkey. Barga and Santos (2010) took an examination on precipitation in Joao Pessoa city, Brazil for various period, 1937-1970 and 1980-1996 and reason that there was a huge yearly precipitation at 10% level. A consistent four-year occasion was seen in 60th's and in the time of 90s of the last century for which it is extremely advantageous for the water stockpiling. Likewise, Mohsin and Gough (2010) made an investigation by utilizing wavelet analysis and the sequential Mann–Kendall test in the Greater Toronto Area. As per their investigation, the urbanization is one of the principal factors for expanding the trend in the yearly mean temperature series and that expanding temperature trend in urban and rural stations started around the 1920s and proceeded until the 1960s. An alternate work has done by Liu et al. (2012). The initially break down the time arrangement to various scale by wavelet strategy. At that point Support Vector Machine was connected to the sub arrangement to simulate and anticipate future conduct and after that, the arrangement was reproduced by the reverse WT, which were the new expectation for the time arrangement. Looking at these two methods they had discovered that forecast accuracy for the new model is higher than that of Support Vector Machine model. The extraordinary local atmosphere is among the fundamental explanation for the breeze sand in bone-dry and chilly zones in Internal Mongolia of China. Zou et al. (2012) in their study concentrated on the time arrangement of temperature

and precipitation in spring. Wind-sand regularly happens on the spring time. With the assistance of Mexican Hat technique for WT, multi-scale variation and unexpected changes in temperature and precipitation of 46 years information amid the time 1959-2004 was investigated. Bhardwaj (2012) utilized Daubechies wavelet and Mexican Hat wavelet for 11 meteorological stations of India-Ahmadabad, Amritsar, Bikaner, Dehradun, Delhi, Gwalior, Hissar, Jaipur, Lucknow, Ludhiana and Shimla amid 2008 to 2010. As per the investigation, with some exception for Shimla, for other station's maximum and minimum temperatures have positive correlation for all of the years under study. On the contrary, all stations for that period under investigation have observed with negative correlation between rainfall and maximum temperature and, rainfall and minimum temperature. As indicated by the authors Karim et al. (2013), WT, through the time-frequency localization, offers better comprehension of the significance of local versus worldwide atmosphere signals. They had four thresholding strategies. From the outcomes it creates the impression that seemingly unrelated regression (SURE) and Minimax give better outcomes as far as Signal-to Noise-Ratio (SNR) and Root Mean Square Error (RMSE) values when contrasted with Heuristic SURE and Fixed-form method. Araghi et al. (2015) utilized the DWT, the Mann-Kendall trend test, and the sequential Mann-Kendall test in their investigation. They applied these techniques on temperature time series at various time scales for the period 1956-2010 in synoptic-scale surface temperatures in Iran and to the predominant time scales influencing these temperature time series and discovered critical positive trend in temperature for month-wise, season-wise and yearly time scales. Again the 2-month and 4-month components were overwhelming at the monthly time scale, the 48 month segment predominant at the seasonal time scale, and the 8-year and 16-year segments prevailing at the yearly time scale. The trend in temperature expanded from west to east in the northern and focal areas of the study zone. In addition, these increasing behavior were most discernible for the seasons-spring and summer. Ideiao and Santos (2005) connected the WT in the monthly rainfall data of northeastern piece of Brazil and got an same outcome as Santos et al. (2001). Caccamo et al. (2015) studied wavelet procedure to meteorological information gathered by methods for

a LSI-LASTEM climate station. They adopted a multi-scale wavelet strategy for the information investigation. The initial segment of the work is routed to the information securing framework portrayal, which likewise incorporates the sensors and the remote microcontroller. At that point, the WT was applied by means of both CWT and DWT. The analysis through wavelet demonstrates that every one of the months there is a positive correlation between maximum temperature and minimum temperature exists while, oppositely to what much of the time happens, a steady negative correlation exists between daily rainfall and maximum temperature as well as daily rainfall and minimum temperature. It was demonstrated that there are significant upward trend for yearly and monsoon rainfall in North Mountainous India and NEI, though negative trend were identified while thinking about India as entirety.

Conclusion

In the study of hydroclimatology, the world wide applications of wavelet analysis are thoroughly increasing which can be noticed by the number of publications of work in the relevant journals in the last two decades. Though a sound knowledge is needed to understand the theoretical concept behind the wavelet analysis, currently the wavelet transforms had turn out to be a very useful statistical tool for studying hydroclimatological time series data.

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