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Markov Chain Model for Studying Disparity in Chickpea Productivity among the Chickpea Producing Districts of West Bengal, India

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ABSTRACT

The main focus of this paper center around the study of disparity in chickpea productivity (kg/ha) among the major districts growing chickpea in West Bengal for the period of 1991-92 to 2020-21. Four districts namely, Nadia, Birbhum, Murshidabad and Malda. By using σ classifier thirty years of chickpea productivity data districts have been classified into three states, namely, highly developed (HD), developed (D) and under developed (UD). Next, observing the transition of the states transition probability matrix and initial probability vectors are constructed for different district. The steady state probability and expected return time to a particular state are also obtained. Here stationary probabilities of different states for different districts under study have been used to predict the future movement of the districts from one state to other state in terms of chickpea productivity. The model developed here is quite general and can be applied in any other study related to disparity for a crop.

Keywords Initial probability vector, Markov chain model, Productivity, Transition probability matrix.

INTRODUCTION

Grain legumes referred as poor man's meat because of its nutritional role in the diet of millions of people in the developing countries. Legumes are vital sources of protein, calcium, iron, phosphorus and other minerals, they form a significant part of the diet of vegetarians since the other food items they consume do not contain much protein (Latham 1997).

The chickpea (*Cicer arietinum*) is an annual legume of the family Fabaceae. It is also called as gram or Bengal gram or channa dal. After dry beans and peas, chickpea is the third most important pulse crop cultivated in the world. Chickpea accounts for 20% of the global pulse production. India ranks first in terms of chickpea production and consumption in the world. In 2020, world production of chickpeas was 15 million tonnes, led by India with 73% of the global total, and Turkey, Myanmar as secondary producers. In 2020-21, India accounts 9.99 million ha area under cultivation and 11.91 million tonnes production with productivity 1192 kg/ha. Chickpea seeds are high in

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protein (20-22%), rich in fiber, minerals, β-carotene and Omega-6 linoleic fatty acid is the major fatty acid present in chickpea oil (46%-62% of total acids) followed by omega-9 oleic acid. Omega-6 fatty acid is one of the essential unsaturated fatty acids for human metabolism that must be incorporated through diet. Chickpea is a key ingredient in Indian cuisine, used in salads, soups and stews, and curry, in chana masala, and in other meal products like channa. Chickpea is grown as winter crop (tropics) and also as summer or spring crop (temperate environments). Legumes crops are often grown as rotation crops with cereals because of their role in nitrogen fixation. Chickpea improves soil fertility by fixing atmospheric nitrogen up to 140 kg/ha.

According Indian institute of pulse research reports 2020-21, in West Bengal chickpea cultivated in 39.77 thousand ha area and 50.07 thousand tonnes of production with 1259 kg/ha chickpea productivity. Among all districts in West Bengal Nadia is the leading district in chickpea production with 5.04 thousand tonnes, which is followed by Murshidabad (15.720 thousand tonnes), Birbhum (16.654 thousand tonnes) and Malda (2.314 thousand tonnes). In chickpea productivity Birbhum stands first with 1444 kg/ha followed by Malda (1345 kg/ha), Nadia (836 kg/ha) and Murshidabad (132 kg/ha). Productivity is the ratio of production of a crop to the area under cultivation of the crop.

Objective of the present paper is to study disparity among the four districts producing chickpea with respect to chickpea productivity (kg/ha) in West Bengal and predict future movement of districts from one state to other state.

Markov model used to model different problems like to show disparity in agricultural production and productivity, prediction of crop yield, stock market shares prediction. Matis *et al*. (1985) used Markov chain approach to demonstrate the development of the forecast yield distributions. Fatokimi and Tanimonure (2021) analyzed current production systems of pigeon pea and forecast the future farm size and examined the constraints to pigeon pea production in Oyo State. Lakshmi and Manoj (2020) compared the performances of five popular stocks using Markov modelling. Disney *et al*. (1988) analyzed the process of structural change in the pork industry using Markov chain model. Sanjay *et al*. (2017) obtained transition probabilities for export markets in India for major pulses and nations importing pulses to India. Shiraganvi and Guledagudda (2017) applied Markov model to export of chickpea from India to different export markets. In this work Markov model has been used to study disparity for the first time.

MATERIALS AND METHODS

A stochastic or random process is defined as a process that changes with the change of time in an unexpected way.

Markov model is a stochastic model used to model randomly changing states over time. Markov process is a particular category of stochastic process where only the current value of a variable is used to forecast the future and the variables in the previous history are considered with very relevant when the current value is given. In most cases, stock share prices are observed to follow a Markov process according to Hull (2018). The MC model's most fundamental property is that the occurrence of state of future event depends only on the current state of variable. The state-space of a Markov process is the set of all possible values that takes. A Markov process whose state space is discrete then termed as Markov chain.

Markov Chain is a particular type of Markov process in which the state of the future events depends on its current state and not on the previous history. Markov process follows memory less property or has a short-term memory.

Memory less property of Markov chain states that the system's state at time (t+1) depends only on the state at time t and not on (t-1), (t-2),…,3,2,1.

Mathematically, the Markov property is stated as

$$P\,(X_{t+1} = x_{t+1} / X_{t}, X_{t-1} = x_{t-1},\ldots,X_2 = x_2, X_1 = x_2, X_1 = x_1, X_0 = x_0) = P\,(X_{t-1} = x_{t-1}/X_t = x_t)$$

for all $t = 0,1,2,\ldots$ and for al the states $x_0, x_1, x_2,.. x_t, x_{t+1}$

Such type of probabilities are called a transition probability.

Transition probability

Probability of transition of the process from state i to state j at 1 step time period, denoted by p_{ij} is defined as,

$$p_{ij} = P(X_{n+1} = j / X_n = i)$$

Probability of transition from state i to state j after k steps time period is denoted by p^k_{ij} where p^k_{ij} defined as

$$p^k_{ij} = P(X_{n+k} = j)/X_n = i) \qquad k \geq 0, \quad i, j = 1,2,3$$

The matrix describing the Markov chain is called a transition probability matrix (TPM).
The probability $p^k_{ij} = P(X_{n+k} = j/X_n = i) \geq 0 \quad k\geq 0 \quad i,j = 1,2,3$ and $n\geq 0$ are the transition probabilities for going to state j from state i in k steps.

Markov chain model is denoted by $\lambda=(A,\pi)$, where A and π are the transition probability matrix and Initial probability vector which are called the parameters of the model.

The possible states of a MC are used as rows and columns, hence the TPM is always a square matrix.

$p_{ij} \geq 0$ for all i and j

Here $A=((p_{ij}))$ $i,j=1,2,3$, where row sum $\sum^3_j = 1\ p_{ij} = 1$
Using σ classifier (to be defined later) the districts have been divided into three states, namely Highly developed (HD), Developed (D) and Under developed (UD).
A TPM for a three state MC is obtained as

$$A = \begin{bmatrix} P(HD/HD) & P(D/HD) & P(UD/HD) \\ P(HD/D) & P(D/D) & P(UD/D) \\ P(HD/UD) & P(D/UD) & P(D/UD) \end{bmatrix}$$

where, P(HD⁄HD) is the probability that districts will be in highly developed state in the current year given that highly developed state has been observed in immediate past year and so on. Thus, symbolically the three state TPM (HD, D, UD) is

$$A = \begin{bmatrix} p11 & p12 & p13 \\ p21 & p22 & p23 \\ p31 & p32 & p33 \end{bmatrix}$$

Since the state space of the present Markov chain model is {HD, D, UD}, therefore the initial probability vector (IPV) consists of three elements p_{01}, p_{02}, p_{03}.

The IPV denoted by ∏_0, is of the form ∏_0=[π_1, π_2, π_3], where for our present study, π_1 is the probability of a highly developed, π_2 is the probability of developed and π_3 is the probability of under developed state for each district.

The equilibrium situation for the data can be obtained by the higher order TPM. Stationary distributions are associated with the Eigen vectors, where the Eigen value is one. To predict the long run behavior of district can be made by using the stationary probability obtained for the districts. Therefore, an attempt has been made to obtain stationary vector for each district under study.

The TPM is

$$A = \begin{bmatrix} p11 & p12 & p13 \\ p21 & p22 & p23 \\ p31 & p32 & p33 \end{bmatrix}$$

The long run behavior of district is observed by determining the higher order TPM of districts as given below

$$A^2 = \begin{bmatrix} p11 & p12 & p13 \\ p21 & p22 & p23 \\ p31 & p32 & p33 \end{bmatrix} \begin{bmatrix} p11 & p12 & p13 \\ p21 & p22 & p23 \\ p31 & p32 & p33 \end{bmatrix}$$

Similarly, calculate the higher order matrix for k step until get stationary vector as [p1 p2 p3].

TPM reaches to the equilibrium state or steady state distribution. In the stationary situation, transition matrix remains stable or invariant if we increase the number of steps.

Markov chain is explained through the diagrammatic representation called the state transition diagram, that is quite same as the TPM but expressed diagrammatically. The state transition diagram of' a Markov chain model λ= (A, π) is a one way directed graph where every vertex depicts the state of the Markov chain model. The parameters of the Markov chain model can well be explained by the state tran-

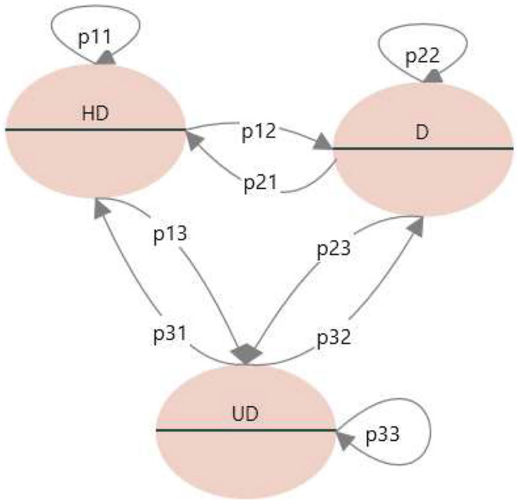

Fig. 1. The schematic diagram of the Markov chain model.

Table 1. Transition of states over the years for different districts with respect to chickpea productivity.

Year	District 1 (Nadia)	District 2 (Murshidabad)	District 3 (Birbhum)	District 4 (Malda)
1991-92	D	HD	HD	HD
1992-93	HD	HD	HD	HD
1993-94	HD	HD	HD	HD
1994-95	HD	HD	HD	HD
1995-96	HD	HD	D	HD
1996-97	HD	HD	HD	HD
1997-98	D	HD	UD	UD
1998-99	D	HD	HD	UD
1999-00	UD	HD	HD	D
2000-01	HD	HD	D	HD
2001-02	HD	HD	HD	HD
2002-03	HD	HD	HD	HD
2003-04	D	HD	HD	HD
2004-05	D	HD	HD	D
2005-06	HD	HD	HD	HD
2006-07	HD	HD	HD	HD
2007-08	HD	UD	HD	HD
2008-09	HD	HD	HD	HD
2009-10	HD	HD	HD	D
2010-11	HD	HD	HD	HD
2011-12	D	HD	HD	HD
2012-13	HD	HD	UD	D
2013-14	HD	HD	HD	D
2014-15	D	HD	HD	HD
2015-16	HD	HD	HD	HD
2016-17	HD	HD	HD	HD
2017-18	D	HD	D	HD
2018-19	HD	HD	D	HD
2019-20	D	HD	D	UD
2020-21	HD	HD	D	D

sition diagram presented in Fig. 1, where the arrow marks represent probabilities of transition from one state to another state.

The steady distribution can also be useful for obtaining the expected return time uij that is the time that the chain visits state j when it left state i. It is reciprocal of probabilities of the stationary vector. Mathematically, the formula for calculating the return time is given as.

$$u_{ij} = \frac{1}{pj}, \; i, j = 1, 2, 3$$

where p' s, j = 1,2,3 are the row elements of stationary vector. Nadia, Murshidabad, Birbhum and Malda districts are ergodic that is TPM reaches to steady state after a large number of transitions. In other words, as k tends to ∞, p^k_{ij} tends to a stationary vector whose rows are identical which is independent of initial state i. This property of limiting distribution of p^k_{ij} is known as ergodicity.

Secondary data for the study purpose collected from Statistical abstract, Evaluation wing, Directorate of Agriculture. Government of West Bengal.

RESULTS AND DISCUSSION

The 30 years Chickpea productivity data of four districts of West Bengal were divided into three states namely HD, D and UD. Without loss of generality let y = x-μ where x is the productivity data having average μ and variance σ^2.

Using σ classifier we divide the districts in the following manner (Table 1)

HD when $-\sigma < y < \sigma$,
D when $-2\sigma < y < -\sigma \cup \sigma < y < 2\sigma$
UD when $-3\sigma < y < -2\sigma \cup 2\sigma < y < 3\sigma$

In order to determine the initial state probability vector, the chickpea productivity data of four districts were divided into three states viz., highly developed, developed and under developed in Table 1. The state space is {HD, D, UD}, and state probability is total number of data in a single state, IPV is denoted by

$\prod_0 = [\pi_1, \pi_2, \pi_3]$.

District 1: Nadia

Initial probability vector for Nadia

Transition of states for chickpea productivity data for 30 years are given in Table 1, where HD = 20, D = 9 and UD = 1.

$\pi_1 = \frac{20}{30}$, $\pi_2 = \frac{9}{30}$, $\pi_3 = \frac{1}{30}$ and state IPV ∏_0 = (0.66667, 0.3, 0.03-3333).

Transitional probability matrix for Nadia

To find the elements of the TPM we count the number of transitions from one state to another state. For example, to find the probability that the district is in HD state given that the district is also in HD state in the immediate past year. We count the number of times the district remains in HD state in the consecutive years. Then that number is divided by the total number years the district was in HD state. Transition probability for HD to HD

$p_{11} = \frac{13}{19} = 0.684211$, similarly calculated remaining elements of TPM $p_{12} = \frac{6}{19} = 0.315789$, $p_{13} = 0$, $p_{21} = \frac{6}{9} = 0.666667$, $p_{22} = \frac{2}{9} = 0.222222$, $p_{23} = \frac{1}{9} = 0.11$-1111, $p_{31} = \frac{1}{1} = 1$, $p_{32} = 0$ and $p_{33} = 0$ respectively.

The state transition probabilities are summarized in matrix form so it is called as Transition probability matrix (TPM).

$$\text{TPM of Nadia} = \begin{bmatrix} 0.684211 & 0.315789 & 0 \\ 0.666667 & 0.222222 & 0.111111 \\ 1 & 0 & 0 \end{bmatrix}$$

Following diagram gives the nature of transition of Nadia district from one state to other state with respect to chickpea productivity Fig. 2.

Transition probability matrix for Nadia district

Nadia	HD	D	UD
HD	0.684211	0.315789	0
D	0.666667	0.222222	0.111111
UD	1	0	0

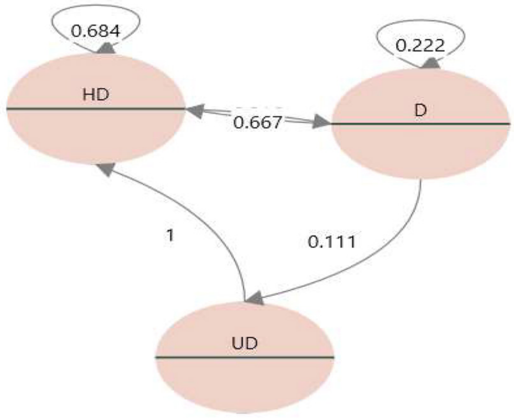

Fig. 2. Nadia transition diagram of Nadia.

Initial transition vector for Nadia district

Nadia	HD	D	UD	Total
π_0	0.684211	0.315789	0	1

After nine years TPM of Nadia district attainted stable or stationary state since 2020-21.

Stationary vector for district Nadia will be reaming in HD, D and UD is [0.689119 0.279793 0.031088] from stationary vector observed that the probability that there is 68%, 27% and 3% likelihood that Nadia district will be in highly developed, developed and under developed category respectively for chickpea productivity in near future and in the long run.

The expected return time to the highly developed state is one year, developed state is three years and under developed state is thirty-two years. Here we have observed that the return time to highly developed state is less as compared to other states and hence we can interpret that highly developed state is occurring more frequently in the process as compared to other states.

Similarly constructed IPV and TPM for rest of districts.

District 2: Murshidabad

Following diagram gives the nature of transition of Murshidabad district from one state to other state with

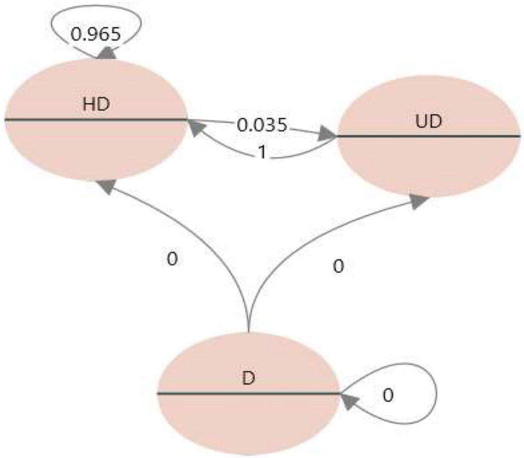

Fig. 3. Transition diagram for Murshidabad district.

respect to chickpea productivity Fig. 3.

Transition probability matrix for Murshidabad district

Murshidabad	HD	D	UD	Total
HD	0.964286	0	0.035714	1
D	0	0	0	0
UD	1	0	0	1

Initial transition vector for Murshidabad district

Murshidabad	HD	D	UD	Total
π_0	0.966667	0	0.033333	1

After five years TPM of Murshidabad district attainted stable or stationary state since 2020-21.

Stationary vector for district Murshidabad will be reaming in HD, D and UD is [0.965517 0 0.034483] from stationary vector observed that the probability of remaining in highly developed state is 96% and 3% in developed state for chickpea productivity in near future and in the long run.

The expected return time to the highly developed state is one year and developed state is twenty-nine years. Here we have observed that the return time to highly developed state is less as compared to other states and hence we can interpret that highly developed state is occurring more frequently in the process as compared to other states.

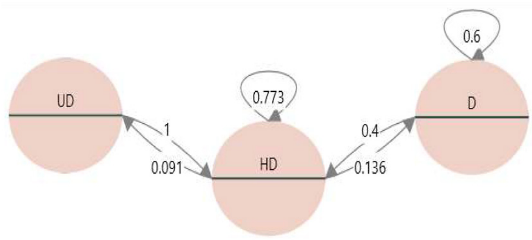

Fig. 4. Transition diagram for Birbhum district.

District 3: Birbhum

Following diagram gives the nature of transition of Birbhum district from one state to other state with respect to chickpea productivity Fig. 4.

Transition probability matrix for Birbhum district

Birbhum	HD	D	UD	Total
HD	0.772727	0.136364	0.090909	1
D	0.4	0.6	0	1
UD	1	0	0	1

Initial transition vector for Birbhum district

Birbhum	HD	D	UD	Total
π_0	0.733333	0.2	0.066667	1

After twenty-two years TPM of Birbhum district attainted stable or stationary state since 2020-21.

Stationary vector for district Birbhum will be reaming in HD, D and UD is [0.698413 0.2380957 0.063492] from stationary vector observed that the probability that there is 69% likelihood that Birbhum district will be in highly developed, 23% developed and 6% in under developed state for chickpea productivity in near future and in the long run.

The expected return time to the highly developed state is one year, developed state is four years and under developed state is fifteen years. Here we have observed that the return time to highly developed state is less as compared to other states and hence we can

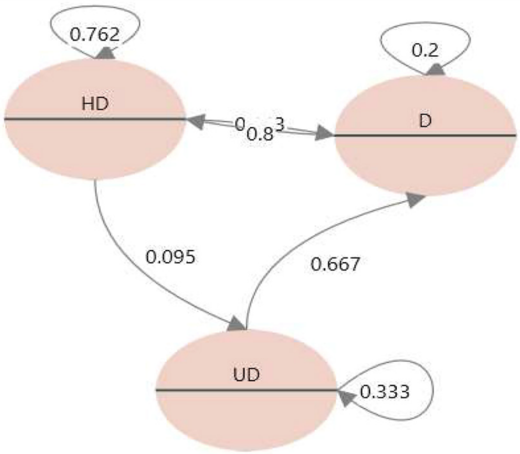

Fig. 5. Transition diagram for Malda district.

interpret that highly developed state is occurring more frequently in the process as compared to other states.

District 4: Malda

Following diagram gives the nature of transition of Malda district from one state to other state with respect to chickpea productivity Fig. 5.

Transition probability matrix for Malda district

Bankura	HD	D	UD	Total
HD	0.761905	0.142857	0.095238	1
D	0.8	0.2	0	1
UD	0	0.666667	0.333333	1

Initial transition vector for Malda district

Malda	HD	D	UD	Total
π_0	0.7	0.2	0.1	1

After eighteen years TPM of Malda district attainted stable or stationary state since 2020-21.

Stationary vector for district Malda will be reaming in HD, D and UD is [0.694215 0.206612 0.099174] from stationary vector observed that the probability that the Malda will remain in HD state in the long run is 69% and that of D state in 20% and UD state will be 9% in respect of chickpea productivity.

The expected return time to the highly developed state is one year, developed state is four years and under developed state is ten years. Here we have observed that the expected return time to highly developed state is less as compared to other states hence it can be said that highly developed state is occurring more frequently in the process as compared to the other states.

CONCLUSION

The model developed here for disparity study of a crop is quite general and can be applied to any other related studies.

It has been found that the chances of districts Nadia, Murshidabad, Birbhum and Malda to remain in HD if the districts were in HD in last year are 0.68, 0.96, 0.77 and 0.76, respectively considering chickpea productivity. Hence the disparity among the four districts with respect to the probability of remaining in HD state in the current year given that the districts were in HD state in last year. The probabilities that districts Nadia, Murshidabad, Birbhum and Malda will remain in D if the districts were in HD in the last year with probability 0.31, 0, 0.13 and 0.14 respectively for chickpea productivity this establishes the disparity among the four districts with respect to the chances of remaining in D state in the current year given that the districts were in HD state in last year has been established. The chances that the districts Nadia, Murshidabad, Birbhum and Malda will remain in UD if the districts were in HD in last year are 0, 0.03, 0.09 and 0.0.09, respectively with respect to chickpea productivity, thus the disparity among the four districts with respect to the chances of remaining in UD state in the current year given that the districts were in HD state in last year is clear.

The chances of districts Nadia, Murshidabad, Birbhum and Malda to remain in HD if the districts were in D in last year are 0.66, 0, 0.4 and 0.8, respectively with respect to chickpea productivity, Hence the disparity among the four districts with respect to the chances of remaining in HD state in the current year given that the districts were in D state in last year is clear. The chances of districts Nadia, Murshidabad, Birbhum and Malda to remain in D if the districts were in D are 0.22, 0, 0.6 and 0.2, respectively considering chickpea productivity. The disparity among the four districts with respect to the chances of remaining in D state in the current year given that the districts were in D state in last year is clear. The chances that districts Nadia, Murshidabad, Birbhum and Malda to remain in UD if the districts were in D in last year are 0.11, 0, 0 and 0, respectively with respect to chickpea productivity, Hence the disparity among the four districts with respect to the chances of remaining in UD state in the current year given that the districts were in D state in last year is clear.

The probability that districts Nadia, Murshidabad, Birbhum and Malda to remain in HD if the districts were in UD in last year are 1, 1, 1 and 0, respectively with respect to chickpea productivity, Hence the disparity among the four districts with respect to the chances of remaining in HD state in the current year given that the districts were in UD state in last year has been established. The probability that districts Nadia, Murshidabad, Birbhum and Malda districts to remain in D if the districts were in UD are 0, 0, 0 and 0.66, respectively with respect to chickpea productivity, the disparity among the four districts with respect to the chances of remaining in D state in the current year given that the districts were in UD state in last year is clear. The probability that districts Nadia, Murshidabad, Birbhum and Malda to remain in UD if the districts were in UD in last year are 0, 0, 0 and 0.33, respectively considering chickpea productivity, Hence the disparity among the four districts with respect to the chances of remaining in UD state in the current year given that the districts were in UD state in last year is clear.

Disparities among the districts with respect to stationary probability of a district to remain in a HD state once after reaching to the HD state can also be established as the stationary probabilities are 0.68, 0.96, 0.69 and 0.69 for Nadia, Murshidabad, Birbhum and Malda respectively. Strictly, speaking the disparity between Birbhum and Malda are not that prominent.

This work also helps in predicting the number of years required by a district to reach the stationary state. It has been observed that Nadia required nine years to reach stationary state. Similarly, Murshidabad required five years, Birbhum required twenty-two years and Malda required eighteen years to reach stationary state respectively.

It has been observed that expected return time to HD state is one year for all four districts. Similarly, expected return time to D state is three years for Nadia, four years for Birbhum and Malda respectively and expected return time to UD is thirty-two years for Nadia, twenty-nine years for Murshidabad, fifteen years for Birbhum and ten years for Malda.

It has been observed that transition took place from D (lower) state to HD (upper) state for some districts the reason may be that famers are adapting improved agricultural technology (better high yielding seeds, improved fertilizer) as the area under the crop has not changed significantly.

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